Re: Standard wreath product & representations

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- *From*: Derek Holt <mareg@xxxxxxxxxxxxx>
- *Date*: Mon, 20 Oct 2008 12:35:20 −0700 (PDT)

On 20 Oct, 16:55, newsgr.m...@xxxxxxxxx wrote:

> Hello,
> I have to put the _standard_ wreath product of C_3 by C_2 into a
> Z_3C_2−module.

I do not really know what you mean. What exactly do you mean by putting a group into a module?

Assuming you are thinking of Z_3 as a finite field and C_2 as a cyclic group of order 2, you can define a Z_3C_2−module of dimension 2, by defining the action of the generator of C_2 on the vector space to be the matrix [[0,1],[1,0]].

If you do that, then the semidirect product of the module by the group will be isomorphic (as a group) to the standard wreath product of C_3 by C_2.

Derek Holt.

> I know that this can be done via coniugation by the
> elements of C_2, but exactly how? (Which is the representation
> needed?)
> [please answer here if the question is clear, continue below if
> not :D]
>
> −−−
>
> Let {1,a,a^2} and {1,b} the underlying sets of (respectively) C_3 and
> C_2. The basis group of W = C_3 wr C_2 is C_3(1) x C_3(b), so W is
>
> (C_3(1) x C_3(b)) ><| C_2*.
>
> For example, a(1) is the permutation of Sym(C_3 x C_2)
>
> (x,1) |−> (xa,1)
> (x,b) |−> (x,b) ;

instead a permutation of C_2* appears like

(x,y) |−> (x,yp)

where p=1* or b*, x in C_3.
The underlying set of B is

{(1(1),1(b)), (1(1),a(b)), (1(1),a^2(b)), ... , (a^2(1),a(b)), (a^2(1),a^2(b))}.

Obviously, by the definition of wreath product, there is an automorphism of C_2* in AutB given by

p* |−> ((c,d) |−> (c,d)^p*). (*)

In view of a possible Z_3C_2−module representation, we have to look for a representation r: C_2 −> GL(B). B is a vector space over Z_3, so that has some sense. Can we assume that r is just a little modification about (*), taking the domain as C_2 and not C_2* ?

Thanks in advance.

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