Re: Radius of largest n+1 balls in n−dimensional unit cube?

Source: http://sci.tech−archive.net/Archive/sci.math/2009−01/msg01472.html

- *From*: quasi <quasi@xxxxxxxx>
- *Date*: Tue, 13 Jan 2009 21:00:23 −0500

On Tue, 13 Jan 2009 17:37:51 −0800 (PST), Matt
<matt271829−news@xxxxxxxxxxx> wrote:

> On Jan 14, 1:17 am, Ross <rmill...@xxxxxxxxxxx> wrote:
>
>> On Jan 13, 1:00 pm, Matt <matt271829−n...@xxxxxxxxxxx> wrote:
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>>>> On Jan 13, 8:35 pm, quasi <qu...@xxxxxxxx> wrote:
>>>>
>>>>> On Tue, 13 Jan 2009 12:18:27 −0800 (PST),
>>>>> Golabi Doon
>>>>>
>>>>> <golabid...@xxxxxxxxx> wrote:
>>>>>
>>>>>> Thank you Quasi! Would
>>>>>> you please provide a hint
>>>>>> about how I can
>>>>>> derive the formula you
>>>>>> proposed? The only piece I
>>>>>> understand is that
>>>>>> sqrt(n) is maximal distance
>>>>>> between two corners of the
>>>>>> cube, but have
>>>>>> no idea how it entered into
>>>>>> your formula.

Ok, but please don't top post. Instead, either bottom post or
intersperse portions of your reply after the relevant parts of the
prior message (as I am doing here). That's the standard in sci.math.

For the configuration I described, r can be calculated as follows:

Place a little cube of edge length r in the corner of the unit cube.
Then the distance from the center of the ball in that corner to the
corner vertex is the length of the diagonal of the little cube, which
is r*sqrt(n) (by proportionality to the unit cube).

Then (draw a diagram to see it),

r + r + r*sqrt(n) = (1/2)*sqrt(n)

which yields

r = sqrt(n) / (2*(2+sqrt(n)))

> On Jan 13, 2:01 pm, quasi <qu...@xxxxxxxx> wrote:
>
>> On Tue, 13 Jan 2009 11:19:11 −0800 (PST), Golabi Doon

<golabid...@xxxxxxxxx> wrote:

Hello,

I would appreciate your help or comment about the following problem. Consider a N dimensional space. If I want to put N+1 balls (all with the same radius R), within the unit hypercube such that:

1. The balls do

not cut through each other
2. One of the balls is at the center of the cube, i.e. at (0.5 , 0.5, 0.5, ..., 0.5)

Then what is the maximum possible R in terms of N? If not easy, a good approximation will be helpful too.

It seems intuitive that the maximum r for your problem occurs when the other n balls are in the corners, tangent to the central ball and tangent to the corner faces. For that configuration,

r = sqrt(n) / (2*(2+sqrt(n)))

But note, the unit cube in R^n has 2^n corners, so you could just as easily have (2^n)+1 balls with the same radius as above, rather than only n+1.

But for n > 4, 4*r > 1, so does that mean they wouldn't actually all fit?

> > The radius is always <1/2, so the central ball always fits in the cube. There is nothing
> > special about r>1/4.
>
> I'm not doubting that the central ball fits in the cube. I'm querying Quasi's claim that immediately preceded my reply −− namely that, in the configuration he describes, you could "easily have (2^n)+1 balls with the same radius" (i.e. a ball at every corner).

And it was a good query.

I take back the claim.

As you point out, since 4r > 1 when n > 4, you can't have balls with
radius r, configured as I described, in 2 adjacent corners.

Thus, for n > 4, you can't have 2^n + 1 balls in that configuration,
with that radius.

As to how many you can have −− good question −− I'll play with it.

quasi
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