# Re: The modern mathematical concept of infinity is indefensible

***Source:*** http://sci.tech−archive.net/Archive/sci.math/2009−01/msg02473.html

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- *From*: Michael Press <rubrum@xxxxxxxxxxx>
- *Date*: Wed, 21 Jan 2009 18:04:54 −0800

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In article
<16457079.1232557274552.JavaMail.jakarta@xxxxxxxxxxxxxxxxxxxxxx>,
"G. Rodrigues" <sorlakind@xxxxxxxxxxx> wrote:

> In article
> <19312788.1232478555757.JavaMail.jakarta@xxxxxxxxxxxxx
> forum.org>,
> "G. Rodrigues" <sorlakind@xxxxxxxxxxx> wrote:
>
>> Do you know the recursion theorem? In rough terms, it states that if you have a rule f such that you know f(0) and given f(n) you can compute f(n + 1) then you can extend f to a function N −> N, N the set of natural numbers. In order to avoid infinite sets, view functions as computer scientists do, as black boxes that given an input spit out an output, so that the recursion theorem just says that given the hypothesis we can compute f(n) for every natural number. "We can compute" means, or can be made to mean, computing in finite time, although in practice finite can be impractically large. This is an idealized situation, of course, but as a self−proclaimed physicist you surely do not object to it since, after all, *all* models of physics and engineering are idealized situations.
>
> Not what I think of as the recursion theorem, even very roughly.

Right, there is a missing sentence there. Let me rewrite the beginning as:

The recursion theorem is a fundamental pillar of computer science since it provides the formal justification for *recursion definitions*. In rough terms, if you have a rule f

> And continue as above. Is this more pallatable?
>
> The correction is moot though, since mr. de Bruijn seems to have chosen not to answer this or any other of mny objections.

Primitive recursive functions are defined. The primitive recursive functions is the smallest class, C, that contains

1) constant functions: lambda xy...z[m]
2) succesor functions: lambda x[x+1]
3) the identity functions: lambda xy...z[w], w = x, y, ...
4) function compostion of prf's.

5) If g(x) and h(x,y,z) are prf's then f satisfying

f(0,x) = g(x)
f(y+1,x) = h(y, f(y,x),z)

is in C.

Here is what I think of when I hear "the recursion theorem."
Let T_1, T_2, ... be an enumeration of Turing machine.

The recursion theorem: If f is a primitive recursive function then there is an n such that

T_{f(n)} = T_n.

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Michael Press
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