Re: .9 repeating

Source: http://sci.tech−archive.net/Archive/sci.math/2009−03/msg02438.html

- *From*: "Jesse F. Hughes" <jesse@xxxxxxxxxxxxx>
- *Date*: Thu, 19 Mar 2009 08:49:04 −0400

lwalke3@xxxxxxxxx writes:

> Indeed, this entire thread has been devoted to seeing
> whether or not there can be a theory in which one can:
>
> 1) Represent positive infinitesimals with digits.
> 2) Find a smallest positive infinitesimal.
>
> Rigorous theories have been found in which either 1) or
> 2) are true, but one in which both is true is lacking.

I'm not sure it's so hard to do both, you know, if you don't require
that *every* "infinitesimal" is represented.

Let's let R' be the set

{f | f: (w + 1) −> {0,...,9}} u {1.000...0}

That is, let R' be the set consisting of "(w + 1) digit sequences"
(this is, according to some, an abuse of the word sequence, but let's
continue).

There's a natural ordering on R', namely the pointwise ordering.

0.000...0 < 0.000...1 < 0.000...9 and
0.999...0 < 0.999...4 < 0.999...9

There's an order−preserving embedding [0,1) −> R', defined as follows:

We map x in [0,1) to the sequence

(x1)(x2)(x3)...0,

where xi is the i'th digit of x, with the proviso that if x has two
different decimal expansions (one ending in all 9s and the other in
all 0s), we use the expansion ending in all 0s.

Now we make an arbitrary choice: we interpret infinite digit sequences (note: here I'm interpreting syntactic objects, not elements of R)

(x1)(x2)(x3)... |−> (x1)(x2)(x3)...0 unless the sequence ends in all 9s
(x1)(x2)(x3)... |−> (x1)(x2)(x3)...9 if it ends in all 9s.

Now according to this, 0.999... is the largest number in R', analogous to the largest element in [0,1). There is a smallest positive number, which is 0.000...1, but I haven't assigned a digit sequence to this number, because I interpreted only omega sequences. If I extend this interpretation (like Mitch?) to the "sequence" 0.000...1, then this guy has a name.

This structure satisfies Mitch's desiderata, more or less.

But it's also completely uninteresting. I don't see why anyone would want to study it at all. I see no reason to believe that analysis done in this structure would be interesting or fruitful and I don't see that this structure matches any of my intuitions about numbers better than classical analysis.

As far as interesting developments, of course, I could be wrong. Maybe there are some keen features of a structure something like this, but I certainly wouldn't invest my time on the hope that such features exist.

I'm sure that this post is chock−full of errors, since I tossed it off as quickly as possible. My point is merely that satisfying (1) and (2) together are neither enough to vindicate MR as a solid mathematical thinker *nor* enough to justify the claim that theories in which (1) and (2) are true are worth pursuing.
−−
Jesse F. Hughes
"Besides, discoverers are too proud to kiss butt. Indiana Jones would never kiss some academic's ass to get published, and neither will I."
−−James Harris
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