Re: A recursion axiom for N?

Source: http://sci.tech−archive.net/Archive/sci.math/2009−03/msg03799.html

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* *Date*: Mon, 30 Mar 2009 20:36:56 −0700 (PDT)

On Mar 29, 8:26 pm, hru...@xxxxxxxxxxxxxxxxxxxx (Herman Rubin) wrote:

> In article <0a6ef43d−a1e0−4b6a−92cb−6f7817546...@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx>,
> Dan Christensen <Dan_Christen...@xxxxxxxxxxxx> wrote:
>
>> After having shelved the problem for a few years, I am back to the
>> problem of proving the existence of recursively defined functions on
>> N, e.g. the "definition" of addition that is usually given:
>> 1. For all x e N, x+1=3Dx'
>> 2. For all x, y e N, x+y'=3D(x+y)'
>> Herman Rubin (Perdue University) has commented here (November 1, 2006)
>> that the existence of these functions does not follow from PA: "One
>> needs a stronger version of induction to do so."
>> From "The Dedekind /Peano Axioms" by D. Joyce, we have:
>> Defining functions by induction, also called recursion. Dedekind and
>> Peano both accepted the principle of mathematical induction for
>> defining functions on the natural numbers N. Both gave arguments that
>> this method should work based on intuitive properties of functions on
>> sets. We'll assume that the recursive definition of functions is
>> valid. It is possible to show that it='s valid using some basic axioms
>> of set theory, or some basic assumptions of logic, but we won't go
>> into that here. Alternatively, this property could be stated as an
>> axiom for N.
>> To define a function f : N −−> S from the natural numbers to a set S,
>> it is only necessary to (1) say what element a of S that f(1) should
>> be, and (2), say what f(n0) should be in terms of n and f(n). In other
>> words, if (1) a is a specified element of S, and (2) g : N : S −−> S
>> is a function that is already defined that takes two arguments, a
>> number k and an element s of S, and yields an element g(k, s) of S,
>> then the new function f can be defined recursively by the two
>> equations
>> (1). f(1) = a, and
>> (2). f(n') = g(n, f(n)).
>> There are, as you can imagine, variants of recursive definition just
>> as there are variants of mathematical induction. The one just
>> mentioned isn't the simpest or the strongest, but it's good enough to

use for many definitions.
Source:http://aleph0.clarku.edu/~djoyce/numbers/peano.pdf
My question to readers here, can it really be shown that, as Joyce put it, "It's valid using some basic axioms of set theory, or some basic assumptions of logic," or must I introduce another axiom in addition to PA for the natural numbers?
Dan

One can either add the axiom of recursive computation, or more simply one can assume that there are addition and multiplication functions satisfying the properties.

Thanks again, Herman!

From a pedagogical perspective, simple sounds good. As a fallback

position, I have been presenting addition and multiplication as "definitions" apart from PA in my program. I see that Peano himself originally included recursive "defintions" of addition and multiplication (really axioms) in his seminal paper. Perhaps I should consider doing the same.

Dan
Download my DC Proof software at http://www.dcproof.com

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those, the Chinese Remainder Theorem is enough to get the rest of inductive evaluation.

Without this, one can prove that, for every x, y, x+y and x*y are defined, but not that the functions + and * are defined. This subtle point seems to have escaped the early workers in the field.

Using the ZF axioms for set theory, it can be shown that the integers defined as the intersection of all inductive sets satisfies the Peano axioms, and furthermore, ordinal recursion is defined in this system, which allows for the definition of the addition and multiplication functions.
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This address is for information only. I do not claim that these views are those of the Statistics Department or of Purdue University.
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