Re: Bound for quotient of eigenvalues

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In article <75dv9iF16ph6tU1@xxxxxxxxxxxxxxxxxx>,
José Carlos Santos <jcsantos@xxxxxxxx> wrote:

> Hi all,
>
> For each real−analytic function _f_ from [0,1] into itself, consider the 2x2 real matrix
>
> a b
>
> b c
>
> where a = int(x^2 f(x),dx), b = int(x f(x),dx) and c= int(f(x),dx) (all integrals are from 0 to 1). Unless _f_ is the null function, this matrix is positive definite and therefore it has two positive eigenvalues. Consider the quotient e_1/e_2 where e_1 is the greatest eigenvalue and e_2 is the smallest one. My question is: is there an upper bound for the quotients obtained by this method? My guess is that the answer is negative, but I was unable to prove it.
>
> Best regards,
>
> Jose Carlos Santos

Take f_m(x) = (m+2)x^m. Then a = (m+2)/(m+3), b = 1, c = (m+2)/(m+1). The matrix M_m −> the matrix of all 1's as m −> oo. The latter matrix has eigenvalues 0 and 2. So you have to expect that for large m, the eigenvalues of M_m are close to 0 and 2, and an easy argument shows this is the case. It follows that e_1/e_2 is unbounded along this sequence.
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