Re: Renewal process

Source: http://sci.tech−archive.net/Archive/sci.math/2009−04/msg01965.html

---

- *From*: Ray Vickson <RGVickson@xxxxxxx>
- *Date*: Sun, 26 Apr 2009 11:18:36 −0700 (PDT)

On Apr 26, 10:57 am, Ray Vickson <RGVick...@xxxxxxx> wrote:

> On Apr 26, 4:54 am, PA1980 <proudasian1...@xxxxxxxxxxxxxx> wrote:
>
> > On Apr 22, 9:27 pm, clvick...@xxxxxxxxx wrote:
> >
> > > On Apr 22, 11:14 am, PA1980
> > > <proudasian1...@xxxxxxxxxxxxxx> wrote:
> > >
> > > > Friend and I are working on this question:
> > > >
> > > > Consider arenewalprocesswith interval
> > > > density
> > > >
> > > > f_X(x) = l^2 x exp(−lx),
> > > >
> > > > defined for x>0. So intervals have a gamma
> > > > distribution (2,l).
> > > >
> > > > a. Delete every event independently with
> > > > probability p. Why is this
> > > > still arenewalprocess? What is the
> > > > newrenewaldensity, and mean
> > > > number of surviving events in an interval of

length v starting from a
surviving event?

If the inter−arrival time density is general (not necessarily a gamma), say with density f(t) [I use 't' for time, instead of 'x'],
then, given that arenewalhas just been "accepted", is the distribution of the time to the next "accepted" arrival independent of
the past? What would the answer to this question say about whether the
"accepted" arrivals yield arenewalprocess? Can you see any role to
be played by a geometric distribution with parameter p? Have you
studied "compound random variables" yet? (These are sums of the form S
= sum{X_i,i=1..N}, where the X_i are iid and N is an integer random
variable independent of the X_i. It is fairly easy to get the Laplace
transform of S in terms of that of the X_i and the
z−transform of N.)
Can you see how to use this to get the new inter−arrival time
distribution, or at least its Laplace transform?

Honestly, I am still stuck. I don't think I understand this process
enough in order to know what to take the Laplace transform?

What about the answer to my first questions. Is the time to the next
accepted renewal independent of the past? What does this say about
whether or not the new process is also a renewal process?

My notes mention something about the distribution of S_n=X_1+...+X_n
having distribution given by integrating a n−fold convolution of f. Is
this what the Laplace transform is for? Honestly I don't even
understand S_n!

If X_1, X_2, ... are the successive inter−arrival times of the
original process, S_n = time until the nth renewal, because it just
adds up all the times between renewal 1, renewal 2, ... , renewal n.
For example, in a Poisson process with rate r, S_n would represent the

time you need to wait for the nth renewal to occur. If you are told you can pack up and leave work after 10 customers have arrived then S_10 would be the time until you can go home.

If f(t) is a density on {t >= 0}, corresponding to a random variable X, then for iid r.v.s X_1, X_2 with density f, the density of S_2 = X_1 + X_2 is the convolution f_2(t) = integral(s=0..t} f(s)*f(t−s) ds; do you see why? (You _really should_ be able to derive this on your own; if not, maybe you are taking a course for which you lack sufficient background.) Anyway, continuing: if X_1, X_2, X_3, ... are iid with denstity f, the density of S_3 = X_1 + X_2 + X_3 = S_2 + X_3 is f_3(t) = int_{s=0..t} f_2(s)*f(t−s) ds [also = int_{s=0..t} f(s)*f_2 (t−s) ds],..., and the density of S_n = X_1 + X_2 + ... + X_n is f_n (t) = int_{s=0..t} f_{n−1}(s)*f(t−s) ds. So, in principle, we can compute the density of S_n. Now, if S = sum{j=0 .. N} X_j, where N is an integer random variable with probability mass function p(n) = P{N = n}, and if N is independent of all the X_j, then the density of S is f_S(t) = sum_{n=0..infinity} p(n)*f_n(t) (with f_0(t) = 0, by definition). For your renewal example, if N = number of arrivals until the next "accepted" one, then P{N = n} = p*(1−p)^(n−1), n = 1,2, ... (a geometric), the time (from the current accepted arrival to the next accepted one) has density given by f_S(t)= sum_{n=1..infinity) p*(1−p)^ (n−1)*f_n(t). So, this is an explicit formula for which you can, in principle, do all the computations. The computations may be very difficult in practice, but that is another issue; it is at this point that something like a transform method may be useful, but never mind that: we are just looking at theory here, not computational details.

I found an example on a Poisson process giving S_n as Gamma distributed.

b. Delte every interval with probability p, so that at the end of each interval there are multiple occurrences.

I cannot even figure out what you are saying here.

I think what's being asked is: delete an interval (instead of the events) so that the events at the end of the interval are coincident. This means the process will have multiple occurrences.

> OK, I think I have it now. We toss a biased coin with P{heads} = p.
> When a "heads" occurs, we put the arriving customer on hold; when the next arrival occurs we toss the coin again and if we get "heads" put the new customer (along with old one) on hold. We continue like this until we get "tails"; then the new customer and all the ones waiting on hold are "admitted" into the facility, or whatever comes after. The time until the next "admission" is just the compound random variable S that I mentioned before. The only difference is that instead of discarding arriving customers (like in the first scenario) we put them into storage. Therefore, the question is asking for the joint distribution of N and S. Can you see how to get that? Finally, you are asked to find the mean number of customers "admitted" in the time interval from 0 to t. I will leave the rest up to you to enjoy struggling with.
>
> R.G. Vickson

I see now that maybe my interpretation here is wrong. Maybe it means the following: _before_ anything happens we toss the coin. If we get "heads" we add a customer (to the next arrival) and toss again. If we get "heads" again, we add a second customer (to the next arrival) and so on. We toss the coins infinitely fast, so no time has passed during all this. Once we get "tails" we stop tossing and adding customers, but just await the next arrival. That arrival will come with its own, personal customer plus all those that we have added by tossing the coin. In _that_ scenario, the time X to the next arrival is just the original gamma (or f(t) in general) and the number of arriving customers is just N, a geometric random variable? Can you see now how to get the joint distribution of X and N? Does X depend on N? Can you see now how to find the number of (actual) arrivals in the interval 0 to t?

R.G. Vickson

> > I am lost on this too.
> >
> > > R.G. Vickson
> > >
> > > > What is the joint
> > > > distribution of the intervals between distinct

event times and their
multiplicities? What is the mean number of
events (including
multiplicity) in an interval of length t from
time 0?

Really, really stuck ...any help would be well
appreciated!

.