Re: Probability, Evolution, and Atheism

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Robert Israel a écrit :

> Alvin Plantinga is a theology professor at Notre Dame, and he wrote this essay on why belief in evolution is irrational:
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> http://www.christianitytoday.com/bc/2008/julaug/11.37.html
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> PZ Myers is a scientist who writes an excellent blog on science and atheism called Pharyngula, and he wrote this entry to refute Plantinga:
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> http://scienceblogs.com/pharyngula/2009/05/alvin_plantinga_gives_philosop.php
> #comments
>
> In his essay, Plantinga uses an example of the extremely low probability of flipping a coin 1000 times, and having heads come up at least 75% of the time.
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> In his refutation, Myers scoffs at Plantinga's innumeracy, wondering why he had to get help to do the calculation: "He needed help from an expert to multiply simple probabilities? Does being a philosopher mean you're incapable of tapping buttons on a calculator?"
>
> Well, I agree with Myers that Plantinga's argument is lame, but I disagree that the calculation is trivial. I would appreciate it if the members of this group would help me out.
>
> The problem is to find the probability that out of 1000 tosses of a fair coin, 750 or more will be heads (I've changed the terminology a bit, but not the math). My math is rusty, but it seems fairly clear to me that the probability of getting *exactly* 750 heads out of 1000 tosses is the number of combinations of 1000 taken 750 at a time, divided by 2 to the 1000th power, i.e.
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> (1000! / (750! * 250!)) / (2^1000)
>
> I hope I'm right so far. Then the probability of getting exactly 751 heads is the same formula, but with 751 at a time, and so on. And so

> the probability of getting AT LEAST 750 heads is the sum of the 250 terms for the probabilities of getting exactly 750, 751, ..., 1000 heads in 1000 tosses.
>
> Am I right so far?

Yes.

> If so, I don't see a non−tedious way to add up the 250 terms, nor do I know of a calculator that you can just "tap buttons" for numbers that size. All of my calculators give an error for any factorial much larger than 70!. Even my Microsoft Math 3.0 won't do combinatorics on numbers that size. I imagine that a specialized program like Maple or Mathematica would, but I don't have those.

Some calculators do have a button for binomial coefficients, which will get you somewhat farther.

> So my question is, is there an elementary formula for summing those 250 individual terms to get the exact cumulative probability, or do you have to use some approximation formula (like Stirling's formula for large factorials) if you don't want to add them one by one? Thanks for any help.

It can be expressed in terms of a hypergeometric function, but that's not "elementary". The usual textbook way of dealing with binomial distribution probabilities for large n is by approximation with the normal distribution, but that's not very accurate in a case such as this, far out into the "tail" of the distribution.

On the other hand, we have things like binomial(1000,751)=249/751*binomial(1000,750), so the number sought is close to binomial(1000,750)*(1+1/3+1/9+...)=3/2*binomial(1000,750), which can be approximated by Stirling. Maple give a true factor of 1.497 instead of 1.5 (and Stirling is correct at 0.1% here) So the prob being estimated is about 0.6 10^−58, and even huge mistakes in the calculation would not change the main result : this probability is *very* small...
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