Re: fourier transform of 1/|x|

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- *From*: David C. Ullrich <dullrich@xxxxxxxxxxx>
- *Date*: Thu, 02 Jul 2009 07:21:33 −0500

On Wed, 01 Jul 2009 11:51:22 −0500, Robert Israel
<israel@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx> wrote:

> David C. Ullrich <dullrich@xxxxxxxxxxx> writes:
>
>> On Tue, 30 Jun 2009 22:45:38 −0700 (PDT), norbert9
>> <norbert9@xxxxxxxxx> wrote:
>>
>>> Indeed, I meant a one−dimensional FT.
>>>
>>> Gradshteyn/Ryzhik (4th ed.) list the FT of 1/|x| as 1/|k| in
>>> 17.23.
>>> I believe it is correct in some sense.
>>
>> In what sense? The most general notion of FT I know is the
>> FT of a tempered distribution, and this is certainly not right
>> in that sense. (Unless, again, they're talking about R^2.)
>>
>>> Mathematica gives
>>> FourierTransform[1/Abs[x],x,k] = (−2 EulerGamma − 2
>>> Log[Abs[k]])/Sqrt
>>> [2 Pi]
>>
>> Do you also believe that
>>
>> 1/|k| = (−2 EulerGamma − 2 Log[Abs[k]])/Sqrt[2 Pi]
>>
>> in some sense?
>>
>>> I'm well aware that \int\cos(kx)/|x| diverges around x=0, but I
>>> suspect there is a way to restrict the support of the function
>>> and

define a meaningful FT.

How?

Perhaps something like this. For eps > 0,

2 int_eps^infty cos(|k| x)/x dx = −2 Ci(|k| eps)
= −2 ln(eps) − 2 gamma − 2 ln |k| + O(eps^2)

To get the Mathematica result as eps −> 0, you have to remove the −2 ln(eps),
which would correspond to the Fourier transform of − 2 ln(eps) Dirac(x).

Of course one's first reaction is something like "fine, but given that we have to 'remove' those things it seems a little wacky to call this a calculation of a Fourier transform".

Second reaction: Hmm, how can we make this into a calculation of an FT?

Say S is the space of Schwarz functions, with dual S', the space of tempered distributions. Say S_0 is the space of Schwarz functions that vanish at the origin.

The dual of S_0 would be (S_0)^* = S'/D, where D is the one−dimensional space of multiples of a delta function. The Fourier transform induces a quotient map from S'/D onto S'/C, where C is the space of constant functions; this is a topological−vector−space isomorphism.

Now 1/|x| _does_ define an element of (S_0)^*, and those truncations converge to 1/|x| in the appropriate weak topology, hence the calculation above does show that the "Fourier transform" of 1/|x| is "1/|k| mod constants".

David C. Ullrich

"Understanding Godel isn't about following his formal proof.
That would make a mockery of everything Godel was up to."
(John Jones, "My talk about Godel to the post−grads."
in sci.logic.)
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