# Re: Is the polynomial ring interpretation of abstract algebra A[X]

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- *From*: "Dik T. Winter" <Dik.Winter@xxxxxx>
- *Date*: Mon, 6 Jul 2009 13:05:47 GMT

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In article <1b2ffbed−40df−4da6−84cb−18b9dd559745@xxxxxxxxxxxxxxxxxxxxxxxxxxx> "Tim Golden BandTech.com" <tttpppggg@xxxxxxxxx> writes:

> On Jul 2, 10:29 am, "Dik T. Winter" <Dik.Win...@xxxxxx> wrote:

....

> > May be. But in a polynomial ring the ring elements are not functions.
> > Addition and multiplication is defined between the elemens of that ring,
> > and those operations make it a ring. It is called a polynomial ring
> > because the elements can be seen as polynomials with coefficients of
> > a base ring.

> You are widening out the discussion here and I appreciate that. I am
> free to compose purely from the ring of reals:
> x − x^3 / 3! + x^5 / 5! − x^7 / 7! + ...

No, you are not free to compose that. You should know that what you write here is not a polynomial but an infinite series, which is a limit in disguise.

> These are a series of elements
> { x,+1!,x,x,x,−3!,x,x,x,x,x,+5!,x,x,x,x,x,x,x,−7!, ... }.
> The redundancy in x can be eliminated if you wish.

What redundancy?

> This function is named sin(x).
> You say:
> "the ring elements are not functions."
> but your distinction is not so strong given that a composition of
> elements with sum and product operators does form a function.

Yes, but as in A[X], X is not an element of A, it does not form a function.

> Particularly we can simply consider the element
> x
> and call it a function, though it's not very exciting.

You can call it that, but in the context of A[X], X is *not* a function.

> Likewise since
> you have already stated that
> x * x
> is an element contradicting your statement above with a function of
> your own claim as one element.

Eh? This makes absolutely no sense to me. If X is an element of a ring,
so are X*X and X*X*X. Otherwise it would not be a ring.

> These are merely the qualities of the definition of ring which we
> discuss. The polynomial form happens to be one compositional pattern
> of the freedoms that exist. Using your previous context we can accept
> any polynomial expression as an element, though I have argued that the
> distinction ought to go more as you have put it here so that
> x * x
> is a product of two elements, that element being assignable to say y,
> but until we form that expression the proper issuance of a new element
> has not actually been made.

Whether it is or is not a new element does not depend on whether we give
it a separate name or not. It depends on the structure of the ring and
the ring operations, that is all.

> This is just quibbling and I don't see that
> much is here to make any provable point on the qualities of X in the
> polynomial A[X].

But X is ony a name of an element that is not in A itself, nothing more,
nor less. So we add to A a new element, call it X. If we want to make
what we have into a ring, we have to define addition and multiplication
between the elements of A and the new element X. This means also that
we get new elements like X+X, X*X, etc. That is the way to form a ring.

> May be, but we can also define a "functional ring" where the elements are functions of a base ring, as addition and multiplication are defined, this also forms a ring. (But it is better to use only functions that are defined everywhere on the base ring.) So with R the base ring, sin(x) and cos(x) are elements of that functional ring and sin(x)*cos(x) is another element, as is sin(x)+cos(x).

Well, this is fine. We can see these as polynomials too.

No, we can't, because they aren't.

> > I wonder if given your statement that R is a subring of C if the same
> > can be proven of X.
>
> What about X? X is not a ring, so it can not be a subring of something, nor can something be a subring of it.

Errr... as I understand it X is being interpreted as
Z is in Q is in R is in C ... is in X .

Clearly you do not understand. X is not a set, so it is not a ring. It is just the name of a new element added to a base ring A.

Oh, I see that notation is bad It's more like
Z is in Q is in R is in C ... is in A[X]

It is not either. Q is *not* in Z[X], because Q is a field and Z[X] is *not* a filed. In general, given a ring A, A[X] is *never* a field, because it does not contain the inverse of X, and so C is *never* in A[X].

There is a little bit of set theory here I need to review.
Like, if
C − R
is the set of complex numbers with its subset of real numbers removed
does this just leave I the imaginary numbers?

No. 2 + i is also in C − R.

Or alternatively is the
R which is a subset of C just the values
x + 0 i

where x is in R?

That is a different question, and that is true.

I believe that this is the correct interpretation, which leaves
x + 1.2 i
in the set C after R is removed, otherwise we could freely substitute
this expression into the products and sums as R, which is clearly
going to break things.

Of course removing R from C does not remove x + 1.2 i for any value of x.
Why should you think that would be the case?

I'm sorry if this digression is not easy to
follow. This could be important to the interpretation of A[X] with A
removed.

Interestingly, (X) is an ideal in A[X], and that is the same as A[X] with
A removed.

Copies of A will still exist in A[X] even when we pull A out of A[X].

No, there are no copies of A still remaining. We know that 0 is an
element of A (because every ring contains a zero). When we remove A, 0
is removed and there is *no* other elements in A[X] with the properties
of 0.

This is consistent with my own arguments earlier.

Perhaps, but because this is nonsense it seems that your earlier arguments
also are nonsense.

> If it is not flawed than at least it grants X some
> qualities beyond a symbol. Beneath here I see you are claiming that
> X X
> is a new element and I see this yet this new element is composed of
> two elements. Until we mark this new element
> c = X X
> then we do not have an element.

Why not? What is in a name? Why would c be a name such that we have a

> > new element and X^2 not? Why are 1, 2 and 12 three different elements of
> > N where I see only 1 and 2?
>
> Please see criticisms above where I see a conflict of interpretation in your thinking here. I know these things are extremely simple that we are discussing, and so frustrating inherently. In that sin(x) is a composition of elements then there is an element which can represent sin(x) if we take the composition as an element. We are breaking down to variables versus constants I believe, which could also be helpful new ground.

This makes no sense to me.

> Upon establishing X as a variable then the ring definition still demands its compositions be in the ring.

But in a polynomial ring X is *not* a variable.

> Whether we label that c
> (5) or sin doesn't matter much. 1, 2, and 12 are radix ten representations above right? If not then I see only two elements with redundancy and notation.

O, I see three different names, whatever those names represent.
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