Re: Fresnel equations and metals

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- *Date*: Tue, 12 Dec 2006 13:25:00 +1000

On Tue, 5 Dec 2006, Enrique Cruiz wrote:

> On 2006−12−05 21:26:15 +0000, "Timo A. Nieminen" <timo@xxxxxxxxxxxxxxxxx> said:
>
>> It might be useful for you if I just post the correct formula for
>> complex refractive index/complex permittivity. I'm conferencing at the
>> moment and am away from office and books. Perhaps next week ...
>
> that would be very useful indeed! Thanks in advance.

OK, here are the results. Explanation of symbols and intermediate
calculations follows the main equations. I recommend that you check that
they give you the correct Fresnel equations when medium 2 is a dielectric
with purely real n_2 (then the angle of refraction gives you k_2z).

Amplitude reflection and transmission coefficients:

TE:

r = (k_1z k_2 Z_2 − k_2z k_1 Z_1) / (k_1z k_2 Z_2 + k_2z k_1 Z_1)

t = 2 k_1z k_1 Z_1 / (k_1z k_2 Z_2 + k_2z k_1 Z_1)

TM:

r = (k_1z k_2 Z_1 − k_2z k_1 Z_2) / (k_1z k_2 Z_1 − k_2z k_1 Z_2)
t = − 2 k_1z k_1 Z_1 / (k_2z k_1 Z_2 + k_1z k_2z Z_1 Z_2)

Power reflection coefficients:

TE:

R = [(k_1z k_2 Z_2 − k_2z k_1 Z_1) (k_1z k_2* Z_2* − k_2z* k_1 Z_1)]
/ [(k_1z k_2 Z_2 + k_2z k_1 Z_1) (k_1z k_2* Z_2* + k_2z* k_1 Z_1)]

= [k_1z^2 |n_2|^2 |Z_2|^2 + |k_2z|^2 n_1^2 Z_1^2

− 2 k_1z n_1 Z_1 Re(k_2z n_2 Z_2)]
/ [k_1z^2 |n_2|^2 |Z_2|^2 + |k_2z|^2 n_1^2 Z_1^2
+ 2 k_1z n_1 Z_1 Re(k_2z n_2 Z_2)]

TM:
R = [(k_1z k_2* Z_1 − k_2z* k_1 Z_2*)
(k_1z k_2* Z_1 − k_2z* k_1 Z_2*)] /
[(k_1z k_2* Z_1 + k_2z* k_1 Z_2*)
(k_1z k_2* Z_1 + k_2z* k_1 Z_2*)]

= [k_1z^2 |n_2|^2 Z_1^2 + |k_2z|^2 n_1^2 |Z_2|^2
− 2 k_1z n_1 Z_1 Re(k_2z n_2 Z_2)]
/ [k_1z^2 |n_2|^2 Z_1^2 + |k_2z|^2 n_1^2 |Z_2|^2
+ 2 k_1z n_1 Z_1 Re(k_2z n_2 Z_2)]

These all assume that n, Z, and k (and all components of k) are purely real in medium 1. They can be complex in medium 2.

k is the wavenumber, k_z is the z component of the wavevector. n is the refractive index, Z is the impedance.

To calculate these:

k = 2 pi n / lambda, where lambda is the free−space wavenumber (ie what you'd have in vacuum).

n = sqrt[1 / (epsilon mu)]
Z = sqrt(mu / epsilon)
where epsilon = permittivity, mu = permeability. For most mediums of interest, mu = mu_0, the permeability of free space, and you can write n = sqrt(epsilon_r) where epsilon_r = epsilon / epsilon_0 is the relative permittivity, also called the dielectric constant.

In medium 1, k_z and k_x can be found from the angle of incidence theta (you can assume k_y = 0).

k_z = k cos(theta)
k_x = k sin(theta)

Trying to do this for medium 2 gets you into the trouble of complex angles if n_2 is complex. Stick to wavenumber and wavevectors.

k_x is the same in medium 1 and medium 2.

To find k_2z, use

k_2z = sqrt(k_2^2 − k_x^2)

and both the real part and imaginary part should be zero or postive (unless you have a medium with gain, or other exotica).

The power transmission coefficient is equal to 1 − power reflection coefficient.

The TE wave is polarised with E parallel to the interface, the TM has H parallel.

RE(..) means the real part.

* means complex conjugate.

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E−prints: http://eprint.uq.edu.au/view/person/Nieminen,_Timo_A..html
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