

Re: I Owe Einstein an Apology. Sorry Albert!

Source: <http://sci.tech-archive.net/Archive/sci.physics.relativity/2004-12/4542.html>

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Excuse me for butting in, but I find so much confusion about the GPS in this thread that I feel the need for clearing up a few misconceptions.

jahn wrote:

> *"The Ghost In The Machine" wrote:*

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>>>>>> *If the clocks are moving then SR predicts they will not remain*

>>>>>> *synchronized.*

Clocks moving in a frame of reference will not generally stay synchronous in that frame, but they may do so in special cases.

For example, clocks moving in a circle with the same speed will stay synchronous to each other in the "stationary frame". They will run slow compared to stationary clocks, but they will stay in sync.

This is the case in the GPS.

All the GPS satellite clocks stay in sync to each other in the ECI frame because they all move in circular orbits with the same speed and at the same gravitational potential.

>>>>> *SR predicts moving clocks can't keep good time?*

>>>>> *I have two garden hoses an egg timer and a bag*

>>>>> *of marbles that says they must be broken.*

>>>>> <http://www.boulder.nist.gov/timefreq/time/commonviewgps.htm>

>>>>

>>>> *The SR correction is $t' = (t - vx/c^2) * \gamma$.*

>>>

>>>> *The vx/c^2 is merely a reflection that the two clocks are*

>>>> *communicating through speed-of-light (e.g., radio), but*

>>>> *the gamma is the killer; $\gamma = 1/\sqrt{1 - v^2/c^2}$.*

The v/c^2 is a reflection of the simultaneity of relativity.
It has nothing to do with "communication through speed of light".

>>>>It's a very slow killer, of course; the typical speed
>>>>of a spacecraft is on the order of $9 \text{ km/s} = 3 * 10^{-5} c$, which
>>>>results in a gamma of about $4.5 * 10^{-10}$. The GPS delta is
>>>>almost exactly this: $4.46 * 10^{-10}$. However, this is at
>>>>best a very very rough estimate, just to give one the feel.
>>>>It's also the wrong sign. :-)

Which should tell you something. :-)

It is actually quite simple to make a first order
calculation of the rate of the GPS satellites.

The relative difference in the rate of a clock in circular orbit
compared to a clock on the surface of the Earth is according to GR
to a first order approximation:

(Approximation of the Schwarzschild solution)

$$(f_2 - f_1)/f_1 = (G*M/(c^2*r_1) - G*M/(c^2*r_2)) - (0.5*v_2^2/c^2 - 0.5*v_1^2/c^2)$$

Where G = gravitational constant, M = mass of the Earth,
 r_1 = radius of the Earth, r_2 = radius of the orbiting clock's orbit,
 v_1 = speed of the Earth clock in ECI frame,
 v_2 = speed of the orbiting clock in ECI frame

Since we have $G*M/r_1^2 = g$, acceleration at Earth's surface, we have:
 $(G*M/(c^2*r_1) - G*M/(c^2*r_2)) = (g/c^2)*r_1*(1-r_1/r_2)$

Altitude of GPS satellites = 20200 km
Orbital period = half sidereal day
Radius of the Earth $r_1 = 6.37*10^6 \text{ m}$
Radius of GPS orbit $r_2 = 26.57*10^6 \text{ m}$
 $g = 9.81 \text{ m/s}^2$

Inserting these numbers, we find that the rate difference
due to gravitation is: $5.28*10^{-10}$ (+45.6 us/day)

So to the speed part:

$$v_1 = 40000\text{km}/(23\text{h } 56\text{m}) = 4*10^7/86160 = 464 \text{ m/s}$$
$$v_2 = 2*\pi*r_2/(11\text{h } 28\text{m}) = 3.87*10^3 \text{ m/s} (-7.1 \text{ us/day})$$

$$0.5*v_2^2/c^2 = 0.83*10^{-10}$$

$$0.5*v_1^2/c^2 = 1.2*10^{-12}$$

Thus the rate difference due to the speed will be: $-0.82*10^{-10}$

The combined rate difference: $(5.28-0.82)*10^{-10} = 4.46*10^{-10}$

Note that the orbiting clock runs fast.

During one day, the difference in proper times will amount to:
 $4.46 \cdot 10^{-10} \cdot 86400 \text{ s} = 38.5 \cdot 10^{-6} \text{ s} = 38.5 \text{ us}$

According to:

<http://vishnu.nirvana.phys.psu.edu/mog/mog9/node9.html>
the factor used in the GPS satellites is $4.4647 \cdot 10^{-10}$.

>>>> *An accurate clock left running for a year in a GPS orbit
>>>> will gain about a hundredth of a second.*

You mean a "normal clock" not slowed down
by the factor $4.4647 \cdot 10^{-10}$?

Yes. It would gain ca. 14 ms a year.

[..]

>>>> *(SR predicts loss
>>>> of time, but then SR requires straight-line freespace
>>>> travel. :-) . Therefore, GPS clocks are "broken" by
>>>> design, coding in this adjustment factor -- and even
>>>> then, they have to be steered from the ground using
>>>> synchronization signals from the TAI.*

1. "GPS time" is a coordinated time where the coordinate system in question is stationary in the ECI-frame. The coordinate time is per definition such that clocks on the geoid will stay in sync (run at the same rate) with this coordinate time. "GPS time" is a theoretical time, derived from all the clocks in the system, that is all the satellite clocks and all the ground station clocks.
2. This "GPS time" is steered so that it (but for a number of whole seconds) is equal to UTC. The spec says it should be within 1 us, but in actual practice, it differs but few ns. This difference is known by the system, and each satellite will transmit the difference GPS-time - UTC so that a receiver can calculate the correct UTC.
<ftp://tycho.usno.navy.mil/pub/gps/utcgps30.dat>
3. The GPS satellite clock are built to run slow (compared to a clock using the SI definition of a second) by the factor $4.4647 \cdot 10^{-10}$ prior to launch. In orbit, they will thus run synchronously to GPS-time.
4. The precision of the clocks is in the order of 10^{-13} . That means that they during one day may drift off sync by several ns - or even tens of ns. The most that can be tolerated is a few tens of ns. (Less than 100ns at the very most)

5. To correct for the latter, ground stations are monitoring the satellite clocks, and are uploading correctional data typically once a day. The "clock offset" is simply how much the clock is off sync. For some satellites, this may be up to milliseconds. The satellite clocks are not corrected, but the "clock offset" is transmitted together with the "clock time", and the receiver calculates the corrected time.

>>>Indeed:

>>><< *GPS TIME STEERING*

>>>=====

>>>*GPS time is automatically steered to UTC(USNO) on a daily basis to keep*

>>>*system time within one microsecond of UTC(USNO), but during the last*

>>>*several years has been within 50 nanoseconds. The rate of steering being*

>>>*applied is +/-1.0E-19 seconds per second squared.*

>>><ftp://tycho.usno.navy.mil/pub/gps/gpstt.txt>

Right.

But note that as stated above. this is steering of the GPS-time, not of the individual satellites.

You have two different coordinated time systems, each "living its own life" which should be made to run equal. The +/- 1.0E-19 seconds per second squared is the maximum "speed" with which the GPS time is changed.

>>>*It must take a REAL good mathematician to get 7 microseconds*

>>>*per day out of that. ;-)*

.. and a real bad physicist to try. :-)

>>*I get 38.53 microseconds per day out of the 4.46×10^{-10} corrective factor.*

>>*It's a simple enough multiplication:*

>>

>> $4.46 \times 10^{-10} \text{ sec/sec} * 86400 \text{ sec/day}$

Right.

> *The altitude effect isn't cumulative.*

Utter nonsense.

> *That is, you can move*

> *from Honolulu to Tibet and make one adjustment to the*

> *length of a pendulum. You don't have to do it every day.*

But the error is still cumulative, isn't it? :-)

> *The 7 ns per day is for transverse doppler effect (less*

> *frequently called the SR correction)*

3 orders of magnitude wrong.
(But we will call this a typo.)

- > *It was included in*
- > *the preset because it does save the receiver some calculating*
- > *(there may have been some political reason to have all*
- > *the "relativistic" effects in one synthezer too [shrug])*
- > *Anyway the 7ns error would accumulate if it represented*
- > *a clock rather than a propagtaion effect.*

Of course a rate error of 7 us/day would accumulate 7 us a day.

- > *I think you can tell at a glance that $\pm 1.0E-19$ steering*
- > *and ± 50 ns / daily won't on the worst day add up to*
- > *the 7ns necessary to attribute to an SR clock anomaly.*

This confusion is too gigantic to try to remedy. :-)
You really have no clue whatsoever, have you jahn Sue?

- > *It's all public information.*
- > http://tycho.usno.navy.mil/gps_datafiles.html
- > <http://tycho.usno.navy.mil/systeme.html>

But you have no idea of what they mean?

Paul