Re: I Owe Einstein an Apology. Sorry Albert!

Source: http://sci.tech−archive.net/Archive/sci.physics.relativity/2004−12/4566.html

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From: The Ghost In The Machine (*ewill_at_sirius.athghost7038suus.net*)
Date: 12/13/04

Date: Mon, 13 Dec 2004 17:01:27 GMT

In sci.physics.relativity, Paul B. Andersen
<paul.b.andersen@deletethishia.no>
wrote
on Mon, 13 Dec 2004 15:28:17 +0100
<cpk8u2$9l5$1@dolly.uninett.no>:
> Excuse me for butting in, but I find so much
> confusion about the GPS in this thread that I
> feel the need for clearing up a few misconceptions.
>
>
> jahn wrote:
>> "The Ghost In The Machine" wrote:
>>>jahn wrote:
>>>>"The Ghost In The Machine" wrote:
>>>>> jahn wrote:
>>>>>>"The Ghost In The Machine" wrote:
>>>>>>>If the clocks are moving then SR predicts they will not remain
>>>>>>>synchronized.
>
> Clocks moving in a frame of reference will not
> generally stay synchronous in that frame,
> but they may do so in special cases.

Ack, bad wording on my part. I meant "moving with respect to *each other*".

> For example, clocks moving in a circle with
> the same speed will stay synchronous to each
> other in the "stationary frame". They will
> run slow compared to stationary clocks,
> but they will stay in sync.
> This is the case in the GPS.
> All the GPS satellite clocks stay in sync
> to each other in the ECI frame because
> they all move in circular orbits with the
> same speed and at the same gravitational potential.

No doubt there are third−order anomalies, hence the steering;
the Earth isn't a perfect oblate spheroid, after all. :−)

>
>>>>>>SR predicts moving clocks can't keep good time?
>>>>>>I have two garden hoses an egg timer and a bag
>>>>>>of marbles that says they must be broken.
>>>>>>http://www.boulder.nist.gov/timefreq/time/commonviewgps.htm
>>>>>
*>>>>>The SR correction is t' = (t−vx/c^2) * gamma .*
>>>>
>>>>>The vx/c^2 is merely a reflection that the two clocks are
>>>>>communicating through speed−of−light (e.g., radio), but
>>>>>the gamma is the killer; gamma = 1/sqrt(1−v^2/c^2).
>
> The vx/c^2 is a reflection of the simultaneity of relativity.
> It has nothing to do with "communication through speed of light".

Hmm...well, if two clocks are spatially separated in an idealized Galilean space, and moving with a velocity v, then the first clock will see the second clock at position x, but the first clock has an observation delay of x/c and the second clock will have moved vx/c.

All these are of course crude first−order approximations.

>
>>>>>It's a very slow killer, of course; the typical speed
*>>>>>of a spacecraft is on the order of 9 km/s = 3 * 10^−5 c, which*
*>>>>>results in a gamma of about 4.5 * 10^−10. The GPS delta is*
*>>>>>almost exactly this: 4.46 * 10^−10. However, this is at*
>>>>>best a very very rough estimate, just to give one the feel.
>>>>>It's also the wrong sign. :−)
>
> Which should tell you something. :−)

Yeah, it tells me I've not studied Hafele & Keating and/or Old Man's equations enough yet. :−)

>
> It is actually quite simple to make a first order
> calculation of the rate of the GPS satellites.
>
> The relative difference in the rate of a clock in circular orbit
> compared to a clock on the surface of the Earth is according to GR
> to a first order approximation:
> (Approximation of the Schwarzchild solution)
>
> (f2 − f1)/f1 =
*> (G*M/(c^2*r1) − G*M/(c^2*r2)) − (0.5*v2^2/c^2 − 0.5*v1^2/c^2)*
>
> Where G = gravitational constant, M = mass of the Earth,
> r1 = radius of the Earth, r2 = radius of the orbiting clock's orbit,
> v1 = speed of the Earth clock in ECI frame,

> v2 = speed of the orbiting clock in ECI frame
>
*> Since we have G*M/r1^2 = g, acceleration at Earth's surface, we have:*
*> (G*M/(c^2*r1) − G*M/(c^2*r2)) = (g/c^2)*r1*(1−r1/r2)*
>
> Altitude of GPS satellites = 20200 km
> Orbital period = half sidereal day
*> Radius of the Earth r1 = 6.37*10^6 m*
*> Radius of GPS orbit r2 = 26.57*10^6 m*
> g = 9.81 m/s^2
>
> Inserting these numbers, we find that the rate difference
*> due to gravitation is: 5.28*10^−10 (+45.6 us/day)*
>
> So to the speed part:
*> v1 = 40000km/(23h 56m) = 4*10^7/86160 = 464 m/s*
*> v2 = 2*pi*r2/(11h 28m)= 3.87*10^3 m/s (−7.1 us/day)*
>
*> 0.5*v2^2/c^2 = 0.83*10^−10*
*> 0.5*v1^2/c^2 = 1.2*10^−12*
*> Thus the rate difference due to the speed will be: −0.82*10^−10*
>
*> The combined rate difference: (5.28−0.82)*10^−10 = 4.46*10^−10*
> Note that the orbiting clock runs _fast_.

Right. Nice piece of work. :−) Saves me a wee bit o' trouble.

>
> During one day, the difference in proper times will amount to:
*> 4.46*10^−10*86400 s = 38.5*10^−6 s = 38.5 us*
>
> According to:
> http://vishnu.nirvana.phys.psu.edu/mog/mog9/node9.html
*> the factor used in the GPS satellites is 4.4647*10^−10.*
>
>>>>>An accurate clock left running for a year in a GPS orbit
>>>>>will gain about a hundredth of a second.
>
> You mean a "normal clock" not slowed down
*> by the factor 4.4647*10^−10?*
> Yes. It would gain ca. 14 ms a year.

Correct. Of course, there are some issues regarding the actual clock manufacture; "fountain clocks" don't work in space, AIUI.

>
> [..]
>>>>>(SR predicts loss
>>>>>of time, but then SR requires straight−line freespace
>>>>>travel. :−)). Therefore, GPS clocks are "broken" by
>>>>>design, coding in this adjustment factor −− and even

>>>>>then, they have to be steered from the ground using
>>>>>synchronization signals from the TAI.
>
> 1. "GPS time" is a coordinated time where the coordinate
> system in question is stationary in the ECI−frame.
> The coordinate time is per definition such that
> clocks on the geoid will stay in sync (run at the same
> rate) with this coordinate time.
> "GPS time" is a theoretical time, derived from
> all the clocks in the system, that is all the satellite
> clocks and all the ground station clocks.
>
> 2. This "GPS time" is steered so that it (but for a number
> of whole seconds) is equal to UTC. The spec says it should
> be within 1 us, but in actual practice, it differs but
> few ns. This difference is known by the system, and each
> satellite will transmit the difference GPS−time − UTC
> so that a receiver can calculate the correct UTC.
> ftp://tycho.usno.navy.mil/pub/gps/utcgps30.dat

Another poster mentioned a delta of 50 ns. That translates into
an accuracy of about 7.5 meters (if one assumes +25 / −25).

>
> 3. The GPS satellite clock are built to run slow
> (compared to a clock using the SI definition of a second)
*> by the factor 4.4647*10^−10 prior to launch.*
> In orbit, they will thus run synchronously to GPS−time.

Like I said, "broken" −− but only because they have to run slow
up in space to compensate for the SR+GR effects. There's not
a lot one can do about it, except, erm, "break" them. :−)

>
> 4. The precision of the clocks is in the order of 10^−13.
> That means that they during one day may drift off sync
> by several ns − or even tens of ns. The most that can
> be tolerated is a few tens of ns.
> (Less than 100ns at the very most)
>
> 5. To correct for the latter, ground stations are monitoring
> the satellite clocks, and are uploading correctional data
> typically once a day. The "clock offset" is simply how
> much the clock is off sync. For some satellites, this may
> be up to milliseconds. The satelite clocks are not corrected,
> but the "clock offset" is transmitted together with the
> "clock time", and the receiver calculates the corrected time.
>
>>>>Indeed:
>>>>< GPS TIME STEERING
>>>>==========

>>>>GPS time is automatically steered to UTC(USNO) on a daily basis to keep
>>>>system time within one microsecond of UTC(USNO), but during the last
>>>>several years has been within 50 nanoseconds. The rate of steer being
>>>>applied is +/−1.0E−19 seconds per second squared.
>>>>ftp://tycho.usno.navy.mil/pub/gps/gpstt.txt
>
> Right.
> But note that as stated above. this is steering of
> the GPS−time, not of the individual satellites.
> You have two different coordinated time systems,
> each "living its own life" which should be made
> to run equal. The +/− 1.0E−19 seconds per second squared
> is the maximun "speed" with which the GPS time is changed.

My brain's beginning to hurt admittedly, but I know that TAI
has some interesting issues (Boulder runs fast); presumably
GPS does, too.

>
>
>>>>It must take a REAL good mathematician to get 7 microseconds
>>>>per day out of that. ;−)
>
> .. and a real bad physicist to try. :−)
>
*>>>I get 38.53 microseconds per day out of the 4.46*10^−10 corrective factor.*
>>>It's a simple enough multiplication:
>>>
*>>>4.46*10^−10 sec/sec * 86400 sec/day*
>
> Right.
>
>> The altitude effect isn't cumulative.
>
> Utter nonsense.
>
> > That is, you can move
>> from Honolulu to Tibet and make one adjustment to the
>> length of a pendulum. You don't have to do it every day.
>
> But the error is still cumulative, isn't it? :−)

If not corrected.

>
>> The 7 ns per day is for transverse doppler effect (less
>> frequently called the SR correction)
>
> 3 orders of magnitude wrong.
> (But we will call this a typo.)
>

> > It was included in
>> the preset because it does save the receiver some calculating
>> (there may have been some political reason to have all
>> the "relativistic" effects in one synthezer too [shrug])
>> Anyway the 7ns error would accumulate if it represented
>> a clock rather than a propagtaion effect.
>
> Of course a rate error of 7 us/day would accumulate 7 us a day.
>
>> I think you can tell at a glance that +/−1.0E−19 steering
>> and +/− 50 ns / daily won't on the worst day add up to
>> the 7ns necessary to atrribute to an SR clock anomaly.
>
> This confusion is too gigantic to try to remedy. :−)
> You really have no clue whatsoever, have you jahn Sue?
>
>> It's all public information.
>> http://tycho.usno.navy.mil/gps_datafiles.html
>> http://tycho.usno.navy.mil/systime.html
>
> But you have no idea of what they mean?
>
> Paul

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--
#191, ewill3@earthlink.net
It's still legal to go .sigless.
```