

## Re: I Owe Einstein an Apology. Sorry Albert!

**Source:** <http://sci.tech-archive.net/Archive/sci.physics.relativity/2004-12/4629.html>

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**From:** Henri Wilson (H\_at\_..(Henri))

**Date:** 12/13/04

Date: Mon, 13 Dec 2004 21:30:39 GMT

On Mon, 13 Dec 2004 15:28:17 +0100, "Paul B. Andersen"  
<paul.b.andersen@deletethishia.no> wrote:

>Excuse me for butting in, but I find so much  
>confusion about the GPS in this thread that I  
>feel the need for clearing up a few misconceptions.  
>  
>  
>jahn wrote:  
>> "The Ghost In The Machine" wrote:  
>>>jahn wrote:  
>>>>"The Ghost In The Machine" wrote:  
>>>>> jahn wrote:  
>>>>>>"The Ghost In The Machine" wrote:  
>>>>>>>If the clocks are moving then SR predicts they will not remain  
>>>>>>>synchronized.  
>  
>Clocks moving in a frame of reference will not  
>generally stay synchronous in that frame,  
>but they may do so in special cases.  
>For example, clocks moving in a circle with  
>the same speed will stay synchronous to each  
>other in the "stationary frame". They will  
>run slow compared to stationary clocks,

Who says?

>but they will stay in sync.  
>This is the case in the GPS.  
>All the GPS satellite clocks stay in sync  
>to each other in the ECI frame because  
>they all move in circular orbits with the  
>same speed and at the same gravitational potential.

They all stay in synch with the ground clock, too, after they are initially corrected for the 'free fall effect'.

.  
>

>>>>>SR predicts moving clocks can't keep good time?  
>>>>>I have two garden hoses an egg timer and a bag  
>>>>>of marbles that says they must be broken.  
>>>>><http://www.boulder.nist.gov/timefreq/time/commonviewgps.htm>  
>>>>>  
>>>>>The SR correction is  $t' = (t - vx/c^2) * \gamma$  .  
>>>>>  
>>>>>The  $vx/c^2$  is merely a reflection that the two clocks are  
>>>>>communicating through speed-of-light (e.g., radio), but  
>>>>>the gamma is the killer;  $\gamma = 1/\sqrt{1 - v^2/c^2}$ .  
>  
>The  $vx/c^2$  is a reflection of the simultaneity of relativity.  
>It has nothing to do with "communication through speed of light".  
>  
>>>>>It's a very slow killer, of course; the typical speed  
>>>>>of a spacecraft is on the order of  $9 \text{ km/s} = 3 * 10^{-5} c$ , which  
>>>>>results in a gamma of about  $4.5 * 10^{-10}$ . The GPS delta is  
>>>>>almost exactly this:  $4.46 * 10^{-10}$ . However, this is at  
>>>>>best a very very rough estimate, just to give one the feel.  
>>>>>It's also the wrong sign. :-)  
>  
>Which should tell you something. :-)  
>  
>It is actually quite simple to make a first order  
>calculation of the rate of the GPS satellites.  
>  
>The relative difference in the rate of a clock in circular orbit  
>compared to a clock on the surface of the Earth is according to GR  
>to a first order approximation:  
>(Approximation of the Schwarzschild solution)  
>  
> $(f_2 - f_1)/f_1 =$   
> $(G*M/(c^2*r_1) - G*M/(c^2*r_2)) - (0.5*v_2^2/c^2 - 0.5*v_1^2/c^2)$   
>  
>Where  $G$  = gravitational constant,  $M$  = mass of the Earth,  
> $r_1$  = radius of the Earth,  $r_2$  = radius of the orbiting clock's orbit,  
> $v_1$  = speed of the Earth clock in ECI frame,  
> $v_2$  = speed of the orbiting clock in ECI frame  
>  
>Since we have  $G*M/r_1^2 = g$ , acceleration at Earth's surface, we have:  
> $(G*M/(c^2*r_1) - G*M/(c^2*r_2)) = (g/c^2)*r_1*(1 - r_1/r_2)$   
>  
>Altitude of GPS satellites = 20200 km  
>Orbital period = half sidereal day  
>Radius of the Earth  $r_1 = 6.37*10^6 \text{ m}$   
>Radius of GPS orbit  $r_2 = 26.57*10^6 \text{ m}$   
> $g = 9.81 \text{ m/s}^2$   
>  
>Inserting these numbers, we find that the rate difference  
>due to gravitation is:  $5.28*10^{-10}$  (+45.6 us/day)  
>

>So to the speed part:

> $v_1 = 40000\text{km}/(23\text{h } 56\text{m}) = 4 \cdot 10^7/86160 = 464 \text{ m/s}$

> $v_2 = 2 \cdot \pi \cdot r_2/(11\text{h } 28\text{m}) = 3.87 \cdot 10^3 \text{ m/s } (-7.1 \text{ us/day})$

>

> $0.5 \cdot v_2^2/c^2 = 0.83 \cdot 10^{-10}$

> $0.5 \cdot v_1^2/c^2 = 1.2 \cdot 10^{-12}$

>Thus the rate difference due to the speed will be:  $-0.82 \cdot 10^{-10}$

>

>The combined rate difference:  $(5.28 - 0.82) \cdot 10^{-10} = 4.46 \cdot 10^{-10}$

>Note that the orbiting clock runs *\_fast\_*.

>

>During one day, the difference in proper times will amount to:

> $4.46 \cdot 10^{-10} \cdot 86400 \text{ s} = 38.5 \cdot 10^{-6} \text{ s} = 38.5 \text{ us}$

>

>According to:

><http://vishnu.nirvana.phys.psu.edu/mog/mog9/node9.html>

>the factor used in the GPS satellites is  $4.4647 \cdot 10^{-10}$ .

>

>>>>>An accurate clock left running for a year in a GPS orbit

>>>>>will gain about a hundredth of a second.

>

>You mean a "normal clock" not slowed down

>by the factor  $4.4647 \cdot 10^{-10}$ ?

>Yes. It would gain ca. 14 ms a year.

>

>[..]

>>>>>(SR predicts loss

>>>>>of time, but then SR requires straight-line freespace

>>>>>travel. :-). Therefore, GPS clocks are "broken" by

>>>>>design, coding in this adjustment factor -- and even

>>>>>then, they have to be steered from the ground using

>>>>>synchronization signals from the TAI.

>

>1. "GPS time" is a coordinated time where the coordinate

>system in question is stationary in the ECI-frame.

>The coordinate time is per definition such that

>clocks on the geoid will stay in sync (run at the same

>rate) with this coordinate time.

>"GPS time" is a theoretical time, derived from

>all the clocks in the system, that is all the satellite

>clocks and all the ground station clocks.

>

>2. This "GPS time" is steered so that it (but for a number

>of whole seconds) is equal to UTC. The spec says it should

>be within 1 us, but in actual practice, it differs but

>few ns. This difference is known by the system, and each

>satellite will transmit the difference GPS-time - UTC

>so that a receiver can calculate the correct UTC.

><ftp://tycho.usno.navy.mil/pub/gps/utcgps30.dat>

>

>3. The GPS satellite clock are built to run slow

- > (compared to a clock using the SI definition of a second)
- > by the factor  $4.4647 \cdot 10^{-10}$  prior to launch.
- > In orbit, they will thus run synchronously to GPS-time.
- >
- >4. The precision of the clocks is in the order of  $10^{-13}$ .
- > That means that they during one day may drift off sync
- > by several ns – or even tens of ns. The most that can
- > be tolerated is a few tens of ns.
- > (Less than 100ns at the very most)
- >
- >5. To correct for the latter, ground stations are monitoring
- > the satellite clocks, and are uploading correctional data
- > typically once a day. The "clock offset" is simply how
- > much the clock is off sync. For some satellites, this may
- > be up to milliseconds. The satellite clocks are not corrected,
- > but the "clock offset" is transmitted together with the
- > "clock time", and the receiver calculates the corrected time.
- >
- >>>>Indeed:
- >>>><< GPS TIME STEERING
- >>>>=====
- >>>>GPS time is automatically steered to UTC(USNO) on a daily basis to keep
- >>>>system time within one microsecond of UTC(USNO), but during the last
- >>>>several years has been within 50 nanoseconds. The rate of steer being
- >>>>applied is  $\pm 1.0E-19$  seconds per second squared.
- >>>><ftp://tycho.usno.navy.mil/pub/gps/gpstt.txt>
- >
- >Right.
- >But note that as stated above. this is steering of
- >the GPS-time, not of the individual satellites.
- >You have two different coordinated time systems,
- >each "living its own life" which should be made
- >to run equal. The  $\pm 1.0E-19$  seconds per second squared
- >is the maximum "speed" with which the GPS time is changed.
- >
- >
- >>>>It must take a REAL good mathematician to get 7 microseconds
- >>>>per day out of that. ;-)
- >
- >.. and a real bad physicist to try. :-)
- >
- >>>I get 38.53 microseconds per day out of the  $4.46 \cdot 10^{-10}$  corrective factor.
- >>>It's a simple enough multiplication:
- >>>
- >>> $4.46 \cdot 10^{-10}$  sec/sec \* 86400 sec/day
- >
- >Right.
- >
- >> The altitude effect isn't cumulative.
- >
- >Utter nonsense.

>  
> > *That is, you can move*  
>> *from Honolulu to Tibet and make one adjustment to the*  
>> *length of a pendulum. You don't have to do it every day.*  
>  
> *But the error is still cumulative, isn't it? :-)*  
>  
>> *The 7 ns per day is for transverse doppler effect (less*  
>> *frequently called the SR correction )*  
>  
> *3 orders of magnitude wrong.*  
> *(But we will call this a typo.)*  
>  
> > *It was included in*  
>> *the preset because it does save the receiver some calculating*  
>> *(there may have been some political reason to have all*  
>> *the "relativistic" effects in one synthezer too [shrug] )*  
>> *Anyway the 7ns error would accumulate if it represented*  
>> *a clock rather than a propagtaion effect.*  
>  
> *Of course a rate error of 7 us/day would accumulate 7 us a day.*  
>  
>> *I think you can tell at a glance that +/-1.0E-19 steering*  
>> *and +/- 50 ns / daily won't on the worst day add up to*  
>> *the 7ns necessary to attribute to an SR clock anomaly.*  
>  
> *This confusion is too gigantic to try to remedy. :-)*  
> *You really have no clue whatsoever, have you jahn Sue?*  
>  
>> *It's all public information.*  
>> [http://tycho.usno.navy.mil/gps\\_datafiles.html](http://tycho.usno.navy.mil/gps_datafiles.html)  
>> <http://tycho.usno.navy.mil/systime.html>  
>  
> *But you have no idea of what they mean?*  
>  
> *Paul*

HW.

[www.users.bigpond.com/hewn/index.htm](http://www.users.bigpond.com/hewn/index.htm)