

Re: variation of appropriate degrees of freedom of metric

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babaluyee@xxxxxxxxx wrote:

In Carroll's text Einstein's equations are obtained from varying the Hilbert action with respect to the full degrees of freedom of the metric, i.e. the metric variation is not restricted to preserving constant signature.

Maintaining constant signature is absolutely essential. But note the variational technique is a CONTINUOUS technique, and both the manifold and the metric are continuous as well. For any given metric, there is a neighborhood in which the signature remains unchanged, and the variation must occur within that neighborhood — this is implicit in using such variational techniques. IOW: the technique is valid only for SMALL variations, and the definition of "small" must include not changing the metric signature.

Particularly, if I try to explore what happens if the matter term in the action is ignored and vary with the full (inappropriate) degrees of freedom of the metric, I obtain solutions that violate the Bianchi identity.

I'm not sure what you mean. If one ignores the matter terms, then the action is just $\int R \sqrt{-g} d^4x$, and varying that gives the (vacuum) Einstein field equation, which clearly satisfies the Bianchi identities, identically. I suspect you made a mistake.

Imagine a boundary condition on $G_{\mu\nu}$ such that it is non-zero in some regions of a spacelike hypersurface then the equation resulting from this variation: $G_{\mu\nu}=0$ outside the boundary would mean the G is not conserved.

You cannot just dictate such a condition on a boundary. There are constraint equations which must be satisfied

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on any Cauchy surface or boundary.

This violation seems to me more a result of varying with respect to inappropriate degrees of freedom of metric than of ignoring the matter term in the action.

Is this correct?

I doubt it. I suspect you tried to use unphysical initial or boundary conditions (i.e. ones which do not satisfy the relevant constraints).

Tom Roberts

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