Re: T−duality

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"Nikolaas Van der Heyden" <nikolaas.vanderheyden@ugent.be> wrote in message news:<20040718121920.DB7FA4C40E9@astra.telenet−ops.be>...

> *I have some difficulties understanding the concept of T−duality in string*
> *theory.*

> *Can You explain me what this means. It must have something to do with the*
> *fact that small scale theories are equivalent to large scale theories, due*
> *to the T−duality.*

Your conceptual understanding is correct. In the simple case of a closed string with one compact dimesion (a circle), there is a critical value for the circle's radius, R*, such that the the spectrum of the theory for R is the same as that for:

(R*)^2/R.

Note that if R > R* then (R*)^2/R < R* and vice versa.

The mapping between the states of the two theories is quite straightforward in this case. The theory with R>R* has states that have energies above the ground state due to the compact dimension for two reasons. The first is that they can have momentum in the compact dimension. These momenta are "quantized" to insure that the wave function for the string is single valued on the circle (usual Fourier series analysis). So there is a sequence of states with ever increasing momenta in the compact dimension.

However, the string also stores energy in the tension of the string. There are a sequence of states for which the string wraps around the circle. These states are also quantized, since a string must wrap an integral number of times around the circle. Each winding adds energy to the state as the string must stretch more, thus increasing the tension.

In general, a string can have an arbitrary combination of momentum and wrapping around the circle, leading to states labeled by two numbers:

$| k, i >_R$

where k is the wave number and i the winding number and the subscript R indicate that this is a state in a universe where the circle has radius R.

Now, what happens when you switch to (R*)^2/R? Well, it is fairly intuitive (if you are used to this sort of thing), that the smaller the radius, the more energetic the momentum modes become (because they are shorter wavelength/higher frequency), and the less energetic the winding modes become (because the string has to stretch less to go around the circle). So it is not implausible that the following two states might have the same energy:

$| k, i >_R \quad |i, k >_{(R^*)^2/R}$

A calculation shows that this is the case: a state in a theory with radius R, wave number k, and winding number i has the same energy as a state in a theory with radius (R*)^2/R, wave number i, and winding number k.

Of course while this is suggestive of a true duality, this is not enough. It is also necessary to show that the dynamics of the two theories are the same. In other words, you need to show that all scattering processes between any set of states is the same in the two theories, after exchanging wave number for winding number.