Re: Covariant Derivative question

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From: Igor (*thoovler_at_excite.com*)
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"Flip Tomato" <flipt@stanford.edu> wrote in message news:<cfjhcpscv1@news.Stanford.EDU>...
> *Greetings−−I'm trying to do some reading into relativistic quantum mechanics*
> *(I've just taken quantum at the undergrad level) and I'm curious about the*
> *covariant derivative that is used when discussing gauge invariance.*
>
> *What motivates the definition of the gauge covariant derivative, other than*
> *that it gives a nice result? I understand the gradient (1) mathematically as*
> *a derivative operator and (2) physically as a momentum operator−−but the*
> *gauge covariant derivative doesn't have any intuitive appeal to me other*
> *than sketchily looking like it would involve the gauge freedom of choosing*
> *A.*
>
> *In Chris Quigg's "Gauge Theories of the..." book, he writes: "Local phase*
> *invariance may be achieved if the equations of motion and the observables*
> *involving derivatives are modified by the introduction of the*
> *electromagnetic field A_\mu(x). If the gradient is everywhere replaced by*
> *the gauge covariant derivative, [this is satisfied]."*
>
> *In terms of the big picture, I understand (but please correct me if i'm*
> *wrong) that local phase invariance is a general principle that we would like*
> *to have in quantum mechanics, so we impose it by introducing this gauge*
> *covariant derivative. The term in the gauge covariatn derivative, A_\mu(x),*
> *then *turns out* to be the EM potential and lo and behold, E&M pops out of*
> *this principle of local phase invariance. The fact that E&M naturally pops*
> *out of this principle−−this is "evidence" to believe that local phase*
> *invariance is a reasonable "first principle"?*
>
> *I know my questions are a little hazy right now as I'm still trying to get*
> *my head around these topics−−but any insight would be much appreciated (and*
> *probably followed by more precise questions).*
>
> *Thanks,*
> *Flip Tanedo*
> *flipt (at) stanford (dot) edu*
>
> *PS−−on a totally unrelated note, I'm not very good with literature searches*
> *yet... how do I find *review* articles in a subject that I'm interested in*

> studying?

The whole point of a covariant derivative, be it in EM gauge theory or in differential geometry as applied to GR, is to literally keep the differential operator covariant. In other words, making sure that the gradient remains a true vector operator under transformations. A true vector must transform in the form v' = M v, where v' and v are column vectors, and M is a square matrix. Quite often, the ordinary derivative operator will not transform in this way and requires additional terms to bring it back in line with the notion of covariance. That's really all there is to it.