Re: Newton's inverse−square force law from Einstein's equation

Source: http://sci.tech−archive.net/Archive/sci.physics.research/2006−02/msg00145.html

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- *Date*: Fri, 10 Feb 2006 18:27:18 +0000 (UTC)

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robert bristow−johnson wrote:

> in article 2LPzf.503845$zb5.17880@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx,
> Richard Saam at rdsaam@xxxxxxx wrote on 01/21/2006 05:53:
>
>> Ted posted this several years ago under the heading
>> "the coolest equation in physics"
>
> i found your 2000 post, Richard, and i have a neophyte question. you wrote:
>
>> Take any small ball of initially comoving test particles in free fall. Work in the local rest frame of this ball. As time passes the ball changes volume; calculate its second derivative at time zero and divide by the original volume. The negative of this equals 1/2 the energy density at the center of the ball, plus the flow of x−momentum in the x direction there, plus the flow of y−momentum in the y direction, plus the flow of z−momentum in the z direction.
>>
>> mathematically, I will interpret this to be:
>>
>> −(1/Volume) d^2 Volume/dt^2 = 1/2 * K * (Energy/Volume
>> + momentumx/(t*areax)
>> + momentumy/(t*areay)
>> + momentumz/(t*areaz))
>> K = 8 pi G/c^2
>
> where does the factor of 1/2 come from? i understand that this 1/2
> factor kills half of the "8" leaving a constant of proportionality of
> 4*pi*G/c^2 which will eventually get us to G*m/r^2 in the classical
> limit instead of 2*G*m/r^2 (which is off by a factor of two). but,

> within the theory of GR, where does that 1/2 come from? it seems to be
> the same question asked in this also old thread:
>
> "Who put the 8PI in the bomp de bomp bomp"
> http://groups.google.com/group/
> sci.physics.research/browse_frm/thread/f7244928ec125ca7
>
> (please unwrap line for URL)
>
> why does that 1/2 have to be there? why does the negative second
> derivative of the ball volume at time zero divided by the original
> volume equal *half* of the energy density at the center of the ball +
> the momentum terms?

Robert

Where does the 1/2 come from?
Here is a neophytic and somewhat circular answer.

Given Newton's second law in relativistic terms:

F = (m*v / (1 − v^2 / c^2)^(1/2)) / dt

let Lorentz transform notation as 'b' :

b = 1/(1 − v^2 / c^2)^(1/2)

therefore:

F = (m*v*b) / dt

Now transform into a work energy relationship
with a K factor of dimensional units 'length/mass'

a*(1/K)*ds = m*b*v*dv

acceleration 'a' can vary and the path 's' is arbitrary
within the dimensional constraints of the equation.

define acceleration 'a' in terms of change in Volume 'V' as:

(d^2 V / dt^2) * (1/V) = constant

and incremental path 'ds' as:

ds = 2*area*dr = 2*4*pi*r^2*dr

'area' is the surface of the spherical volume 'V'

Now work−energy relationship becomes:

(d^2 V / dt^2) (1/V) *(1/K)*2*4*pi*r^2*dr = m*b*v*dv

Now integrate both sides:

(d^2 V / dt^2) (1/V) *(1/K)*2*(4/3)*pi*r^3 = −(m*c^2 − m*v^2)*b

for v << c then b ~ 1
and V = (4/3)*pi*r^3

then:

−(d^2 V / dt^2) (1/V) *(2/K)*V = (m*c^2 − m*v^2)

and:

−(d^2 V / dt^2) (1/V) *(2/K) = (m*c^2/V − m*v^2/V)

and:

−(d^2 V / dt^2) (1/V) = (K/2)*(m*c^2/V − m*v^2/V)

and:

−(d^2 V / dt^2) (1/V) = (K/2)*(m*c^2/V − m*v^2/V)

and:

−(d^2 V / dt^2) (1/V) = (K/2)*(m*c^2/V − m*v/(areax*time)
− m*v/(areay*time)
− m*v/(areaz*time))

This is the same as the Baez narrative interpretation
except for the negative signs on momentum.
Since momentum 'm*v' is a vector
and energy 'm*v^2' is a scalar

perhaps the relationship should be expressed as:

−(d^2 V / dt^2) (1/V) = (K/2)*(m*c^2/V +/− m*v/(areax*time)
+/− m*v/(areay*time)
+/− m*v/(areaz*time))
or

−(d^2 V / dt^2) (1/V) = (K/2)*(m*c^2/V +/− m*v/arear)

or perhaps in terms of spherical surface area 'arear'

−(d^2 V / dt^2) (1/V) = (K/2)*(E/V +/− m*v/(arear*time))

Here is a neophytic answer for the '2' factor would be
that is left after expressions for volume 'V'
and spherical surface area '4*pi*r^2 ' are considered.

Lets look again at the relationship:

a*(1/K)*ds = m*b*v*dv

lets define another path 'ds'

ds = 16*pi*r*dr

a = d^2 r / dt^2 = constant

then

a*(2/K)*8*pi*r*dr = m*b*v*dv

integrate

a*(2/K)*4*pi*r^2 = −(m*c^2 − m*v^2)*b
= −(m*c^2)/b

K = 8*pi*G/c^2

and solve for 'a'

a = −(G * m / r^2)/b

or

a = −(G * m / r^2)*(1 − v^2 / c^2)^(1/2)

Does this equation correctly reflect
the relativistic gravitational acceleration
such as a satellite orbiting the earth??

Of course this equation reduces to the standard
Newtonian gravitational acceleration at 'v << c'

a = −(G * m / r^2)

And again the '2' factor appears to be
a 'left over' consequence of considering
the spherical surface area '4*pi*r^2 '

The relationship:

a*(1/K)*ds = m*b*v*dv

would appear to be general in nature

and be an embodiment of the equivalence principle.
Various candidate accelerations 'a'
and geometric paths 'ds' could be analyzed
within dimensional constraints of this equation.

Somebody back in the early days,
must have done this 'simple' type of reasoning.

Richard

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