

Re: Fun problem

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- *From:* "Jay R. Yablon" <jyablon@xxxxxxxxxxxx>
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The below was never posted, sent about 4–5 days ago. Jay.

"Igor Khavkine" <igor.kh@xxxxxxxx> wrote in message
<news:1145745718.411724.269370@xx>

I recently ran into a fun problem from an old Russian physics olympiad. I found the solution a little surprising and intriguing. The problem requires no more than high school classical mechanics and some ingenuity. Here it is for your amusement.

Consider a point particle sliding on a flat table (ignore friction). The table has a cylindrical hole of finite depth (vertical walls, flat bottom). The particle can approach the hole with different velocities and with different impact parameters (the particle's motion need not be directed toward the center of the hole). As the particle falls into the hole, it starts bouncing off the walls and the bottom (assume elastic collisions). Sometimes it gets stuck in the hole forever, sometimes it escapes (bounces out). Determine the relation between the depth of the hole, its radius, the particle's initial velocity, and impact parameter necessary for the particle to escape after it falls in.

Enjoy!

Igor

Initial impression of how to approach this:

Start by looking at the case where the particle goes straight to the center of the hole (direct impact parameter — great putt!). If the hole has a radius r , the particle will jump back out if its velocity is such that it strikes the bottom of the hole at the exact center, because its upward path after hitting the bottom will precisely reflect the downward path due to the perfect elasticity assumption. It will bounce out at the opposite side of its entry, and continue forward.

The next way for it to escape is to hit $1/3$ of the way across. It will then bounce up and reach escape height $2/3$ of the way across, and hit bottom again the whole way across. Then, it will precisely reverse

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trajectory and come out the same way it came in, and continue back toward the golfer. (Sounds like my putts.)

At $1/4$, it will escape after a second bounce at $3/4$, and move forward on escape. At $1/5$. escape but backwards. $1/6$, escape but forward, etc. $1/n$ where n is even is forward escape. $1/n$ where n is odd is backward escape.

Now, let's look at $2/3$. Hits bottom at $2/3$, $6/3$, $10/3$, $14/3$ etc. Hits top at $4/3$, $8/3$, $12/3$. $12/3=4$ is the answer. Goes back and forth twice (four traversals), then escapes backwards.

Next, $3/4$. Bottom at $3/4$, $9/4$, $15/4$, $21/4$. Top at $6/4$, $12/4$, Forward escape after three traversals.

Next, $4/5$. Top (that what counts) at $8/5$, $16/5$, $24/5$, $30/5$. Bingo. Six traversals, backward escape.

Next, $3/5$. Top at $6/5$, $12/5$, $18/5$, $24/5$, $30/5$. Again, six traversals, backward escape.

Next, $2/5$. Top at $4/5$, $8/5$, $12/5$, $16/5$, $20/5$. Backward escape, four traversals. We did $1/5$ before.

Next, $5/6$. Top at $10/6$, $20/6$, $30/6=5$ traversal, forward escape.

General rule: escapes if velocity, radius and depth (and the gravitational acceleration) are such that it strikes the bottom m/n of the way across the hole, where m and n are integers. If n is even it is forward escape. If n is odd it is backward escape.

The impact parameter at first thought does not change anything: it still needs top strike m/n of the way across the hole on whatever path it takes toward the hole. This means that for a less than square impact, the velocity will need to be reduced proportionally to the length of hole over which it travels. BUT, on second thought: when it hits the hole wall, it will not bounce straight back any more. It will go at an angle. I suspect that you will then also need to consider paths which are triangles, squares, pentagons, etc. That how I'd lay this out. The quantization m/n will carry over to the angles of incidence, which will have similar quantum constraints. That is step 2 of this problem. The interesting result here will be the angles at which it bounces out.

One should also consider the extreme case of an exact tangential graze. Then, as I think was pointed out earlier, it will spiral in, hit bottom, then bounce back out. The exit angle will depend on depth, radius, and velocity.

After all of this is laid out, then one reverse engineers the math to get the formula involving the "depth of the hole, its radius, the particle's initial velocity, and impact parameter," assuming, e.g. a

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sea-level gravitational acceleration on earth at the equator, or some such thing.

Jay.

Jay R. Yablon
Email: jyablon@xxxxxxxxxxxxx