Re: What exactly is wrong with Huygens' principle in two dimensions?

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In article <1166344055.124563.258610@xxxxxxxxxxxxxxxxxxxxxxxxxxxx>,
<frank_k_sheldon@xxxxxxxxxxx> wrote:

> But even those pages have pictures where the principle is
> illustrated in two dimensions. (As do almost all books and websites.)
> The enevelope of waves behind a ship is also often deduced in
> this way, and that is a purely 2−dimensional effect.

In any dimension, the *envelope* a pattern of waves satisfying
the wave equation can be deduced by drawing a circle of radius
t/v about each point that emitted a wave at a time t ago, if
the waves move at velocity v. This "weak" version of Huyghens'
principle works in any dimension.

The full−fledged Huyghens principle says that if a point emits
a wave at a time t ago, there will be no wave anywhere *except*
at the circle of radius t/v about that point. This works only
when the dimension of space is odd and greater than 1.

> This leads to 2 issues:
>
> (1) What exactly does not work in two dimensions, given that
> all drawings to explain the principle are 2−dimensional?
>
> (2) Is there a two−dimensional wave effect that one CANNOT understand
> with Huygens' principle?

We can give the same answer to both questions. First consider
3 dimensions. If a point source of light blinks on for an
instant at time 0, at some later time t there'll be no light
visible except right on the surface of the sphere of radius t/c
centered at this point. This is the strong version of Huyghens'

principle.

Next consider the same situation in 2 dimensions. We can figure out what happens using 3−dimensional reasoning, since a point source of light in 2 dimensions acts exactly like a *line* source of light in 3 dimensions!

Using the 3d Huyghens principle together with the superposition principle, we see that a point at a distance t/c from the line source will *first* see light at time t. But, it will continue to see light at later times, emitted from points further away along the line. So, it will see a decaying "afterglow" after the initial burst of light.

In short: since the strong version of Huyghens' principle holds in 3 dimensions, it cannot hold in 2 dimensions.

And, there's nothing special about the numbers 2 and 3 here. The same argument shows this: if the strong version of Huyghens' principle holds in n−dimensional space, it cannot hold in (n−1)−dimensional space.

So: since the strong version of Huyghens' principle holds in 3−dimensional space, it cannot hold in 4−dimensional space.

We can't draw any further conclusions about Huyghens' principle in various dimensions from what I've said so far. But, explicitly solving the wave equation for a point source of light shows that the strong version of Huyghens' principle holds in ODD dimensions greater than 1, but not EVEN dimensions or − the curious exception − 1 dimension. This is consistent with the fact that if the principle holds in n−dimensional space, it cannot hold in (n−1)−dimensional space.

All this stuff was worked out in ancient discussions on sci.physics.research... I forget by whom.

By the way − the Dutch write "Huyghens", but NASA has taken to writing "Huygens".

Happy Holidays!

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Puzzle 33: Who owns all the unmarked mute swans on the River Thames?

If you get stuck, see

http://math.ucr.edu/home/baez/puzzles/33.html

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