Re: confused w/ decoherence

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- *From*: Stephen Parrott <S_Parrott@xxxxxxxxxx>
- *Date*: Wed, 28 Mar 2007 18:00:18 +0000 (UTC)

> [mahdiant@xxxxxxxxx writes:]
>
> In the density matrix formalism,
> it's said that after a measurement of
> an observable A = Sum (a_i P_i),
> with P_i projection operators and a_i eigenvalues of A,
> a system which was originally described
> by a density matrix rho
> is now described by rho' = Sum P_i rho P_i.
> This states, among many things,
> that a pure state can become a mixed state by means of a measurement.
> What I don't understand is as follows:

That's all correct. A measurement can convert
a pure state to a mixed state in the way that you describe.

> [...]
>
> But what about a single particle?
> The density matrix is equally well used
> for an ensemble and a for part of a big system
> (when the system has a state vector but the part doesn't).

I assume you mean something like the following.
We have a system
(maybe a single particle, maybe something more complicated)
which if treated as an isolated system would be studied
as described in beginning quantum mechanics texts.
For example, evolution through time t
of a pure state f would be given by

(1) f −−−> U(t)f , where U(t) is a unitary operator.

And time evolution of a mixed state S
(represented by a positive Hermitian operator of trace 1)
would be given by

(2) S −−−> U(t) S U(t)* , where * denotes adjoint.

[In the following, I will assume for simplicity that
all Hilbert spaces appearing are finite dimensional,
so that we don't have to worry about technical problems
in infinite dimensions, such as existence of traces.]

That elementary quantum mechanics approach is similar to
studying Newtonian mechanics assuming that there are no
frictional forces. This approach works well for celestial mechanics,
but not so well if we are studying the motion
of a creaky old wagon that needs lubrication.

Similarly, the idealizations of elementary quantum mechanics
may not well describe the behavior of "real−world" quantum systems.
A commonly used attempt to introduce "quantum friction" (more formally
known as "decoherence") into the mathematics is the following.

Assume that the "ideal" system which we are studying
(in your case, a single particle) is coupled to an "environment".
The environment is a separate quantum system which
is regarded as not fully known,
just as frictional forces are usually too complicated to
be described in full detail.

The pure state space for two quantum systems
(our "ideal" system and the "environment")
joined together to form a single larger quantum system are
traditionally described as the tensor product of the pure state spaces
for the two systems. Also, the full (mixed) state space of the
larger system is the tensor product of the individual (mixed) state
spaces.

In the following, by "mixed state", I will mean a state
which is not necessarily a pure state (but can be).
It is described by a positive Hermitian matrix of trace one.
(Warning: Don't confuse this with a matrix with positive elements.)
If it is a pure state, this matrix is a projector
with one−dimensional range.

Given a (mixed) state L of the larger system,
what is the corresponding state of the original "ideal" system?
Traditionally, it is assumed to be the PARTIAL TRACE of L
over the environment. (There are reasonable arguments justifying
the assumption.) If L happens to be a product state,
which means that

L = S tensor E,

with S a state of the "ideal" system and E a state of the "environment", then the partial trace of L over the environment is just S, which is what we would expect. But not every state in the tensor product of two systems is a product state (a general state is a sum of product states), so the general situation is more complicated.

Suppose we start with the above product state L and let it evolve through a time t. Then, since the rules of elementary quantum mechanics are assumed valid for the larger system (that's like assuming that if we knew all frictional forces exactly, then Newtonian mechanics would exactly describe the motion of the creaky wagon), the resulting state is:

V(t) L V(t)* = V(t) (S tensor E) V(t)* ,

where V(t) is a unitary operator on the larger system.
(This is not necessarily a product state.)

The corresponding final state S' of the original "ideal" system is the partial trace over the environment of this,

S' = partial trace (V(t) (S tensor E) V(t)*),

and that partial trace IS NOT NECESSARILY OF THE FORM

U(t) S U(t)* ,

where U(t) is some unitary operator on the original "ideal" system.

If S' is not of the form U(t)SU(t)*, then what is it?
Calculation shows that the answer is that it is of the form

(3) S' = Sum_i M_i S M_i* ,

where M_1, M_2, ... , M_n are matrices satisfying the condition

Sum_i M_i* M_i = identity matrix.

This is a trace−preserving operation, but not necessarily of the form
S' = U(t) S U(T)*.

[This calculation is carried out for the case of a projector E in the fine book of Nielsen and Chuang, "Quantum Computation and Quantum Information", p. 360, eq. (8.10), and the general case is a simple extension (a specialization of eq. (8.34), p. 364).

But be warned that their discussion introduces for the first time and without explicit mention an unusual notation. If the reader incorrectly guesses its meaning (as I did on first reading), the calculation may look like nonsense.]

The bottom line of this discussion is that if we mathematically treat "quantum friction" as described above, then the rules of elementary quantum mechanics no longer give the time evolution of the original "ideal" system which we set out to study. In particular:

(i) A pure state S can evolve into a non−pure state (3).

(The rules (1) or (2) of elementary quantum mechanics preclude this.)

(ii) Time evolution is no longer necessarily given by a unitary equivalence S −−−> U(t) S U(t)*.

That doesn't quite answer the question you seemed to be asking, but it sets up a background putting the answer in context. Now let's turn to the question itself.

> For a single particle, traditional experiments (double slit, etc.) indicated that the successive experiments' results depend on the previous ones'.

[This is a parenthetical remark not directly relevant to the main discussion. It is true that the mathematics implies that "successive experiments' results depend on the previous ones", but I'm not aware of any clean experimental tests of this in a simple context (e.g, double slit). If you have any references, I'd like to look at them.]

> Nevertheless, it seems to me that the decoherence approach (devised to get rid of the wave function reduction) states that a measurement (even on a single particle, not an ensemble) is just the interaction of the system with the environment in such a way to convert a pure state to a mixed one as above. So how can then the result of a further measurement depend on the outcome of the previous one?

Let's take a simple example. Suppose the measurement is binary, i.e., has only two possible results, 0 and 1.
In elementary quantum mechanics, this is described as follows.
There is a projector P corresponding to result "1", and (I−P) corresponds to "0" (I := identity).
If the system is in mixed state S before the measurement, then after

it is in state

(4) P S P + (I−P) S (I−P) .

In particular, if the density matrix S represents a pure state,
then the post−measurement state (4) is usually a non−pure state
as you say (the exception being when S commutes with P, in which case
the measurement doesn't change S).

Now suppose there is "quantum friction" necessitating
describing the situation as above, i.e. the original system is
tensored with an "environment" to obtain a larger system to which
elementary quantum mechanics applies. If the larger system
is in state L, then the state S of the original system is
the partial trace over the environment of L.

Suppose the larger system starts in a product state

L = S tensor E ,

with S a state of the original system, and E a state of the environment.
Next perform a binary measurement on the larger system.
This corresponds to the replacement

(5) L −−> Q L Q + (I−Q) L (I−Q),

where Q is a projector. Finally, obtain the final state S' of
the original system by partial tracing over the environment the
right side of (5). The result is not obvious,
but calculation (Nielsen and Chuang, p. 364, eq. (8.32) and (8.34)
with their U := I) yields:

(6) S −−> S':= partial trace (QLQ + (I−Q)L(I−Q) = Sum_i M_i S M_i*,

where the M_i are matrices satisfying

Sum_i M_i* M_i = I .

This not usually of the form (5).
One big difference between (5) and (6) is that applying (5) twice
is the same as applying (5) once, but applying (6) twice need not
give the same result as applying (6) only once.

Your question is a good one.
I hope that answers it.

Further remarks:

(a) It seems disconcerting that it should be necessary
to abandon the rules of elementary quantum mechanics
to describe non−ideal systems. Maybe the rules of

elementary quantum mecanics should be changed to
apply directly to non−ideal systems (e.g., replace
(2) by (3) and (5) by (6)). Then both time evolution
and measurement would be described by the same mathematical
transformation (since the right sides of (3) and (6)
are the same), a pleasing conceptual simplification.
And, this still does not rule out unitary time evolution
in "friction−free" situations.

(b) The above refers often to the fine book of Nielsen and Chuang,
so perhaps I should warn that although it is *much* clearer
than the average physics text, it does make significant demands
on the reader.

It assumes a fine mastery of abstract linear algebra
and quantum mechanics in finite dimensions.
That might seem simple, but what passes as
an applied linear algebra course these days
(mainly almost useless matrix manipulations)
is nowhere near enough. Also, I've heard that usual
undergraduate courses in quantum mechanics
pay little attention to subtleties of the density matrix formalism,
which is critical background for this book.
A blurb on the cover claims that it is "accessible to anyone with a good
undergraduate background in math, computer science, or
physical sciences", but I would change that to
"an EXCEPTIONALLY good undergraduate background
in math, computer science, AND physics".
I suspect that most undergraduates may find
much of it hard going.

I've just finished reading this book cover to cover,
and I was puzzled for a long time by their introduction of
an untraditional axiom for what they call "measurement operators".
Indeed, I puzzled over this until I got to page 500 and something,
when I realized that they are basically the M_i discussed above,
and that Nielsen and Chuang are partly changing
the rules of elementary quantum mechanics as suggested above in (a).
But since they only partly change them, I'm not sure
that the resulting treatment is fully consistent.
For example, they replace (5) by (6), but not (2) by (3).

I would advise potential readers of this book to initially ignore
the "measurement operators". They are hardly used anyway.
The traditional treatment in which the M_i are orthogonal projectors
is sufficient for 99% of the book.

(c) I am preparing a review of this book to join other reviews on
my website www.umb.edu/~sp. It will discuss in more detail some
of my reservations about it. Interested or potential readers might

find it there in a week or two. I also hope to post a page of reviews of interesting papers which has been awaiting completion for a long time.

In general, this is a really good book.
It is in the same class as Misner, Thorne, and Wheeler's "Gravitation", and Mackey's "Mathematical Foundations of Quantum Mechanics".
It has been decades since I have come across a physics book this good, despite some reservations.

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