Re: U(1) and SU(2) as subgroups of SU(3)

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Hello Alex:

It would be interesting how they animate SU(3). Are they available

on the web somewhere?

The URLs are at the end. The first one is a collection of the other
5.

Since SPR is text based, I'll provide a few highlights.

I like the animation of U(1). In one complex plane, U(1) is a circle. With three complex planes, U(1) can be tilted in all three, appearing as an ellipse in all three planes. In space, the two points move along a straight line. What changes is the velocity.

The group SU(2) doesn't look like an animation I have ever seen before. The algebra is simple, just 4000 random quaternion plugged into exp(q − q*). The subtraction gets rid of the first term, and the exponential has a norm of one. There are few points where t is less than zero (they do exist though). The point eventually form a sphere that shrinks. You'll have to play the animation to understand.

The group U(1)xSU(2) looks like an expanding, then contracting sphere. It has a strong bias for events where t is negative. The algebra I used was based off of SU(2). I calculated q/|q| exp(q − q*). Although quaternions do not commute with other quaternions in general, they do commute with themselves. I use the same quaternion throughout, so there are 1+3 degrees of freedom.

I was wondering how to construct a representation of the group SU(3). I knew its Lie algebra has eight generators. As a group it must have an identity, every element must have an inverse, and to be part of SU(3) the norm had to be equal to one. The group multiplication table had to be different from U(1)xSU(2). I decided to toss in a conjugate, and calculate this:

(q/|q| exp(q − q*))* q'/|q'| exp(q' − q'*)

Now I am using two quaternions which have 8 degrees of freedom. This
is still a division algebra, so there is an identity (1, 0, 0, 0) and
every element has an inverse. The product is non−associative, because
a* (b c) != (a b)* c.

The animation is distinct. The group SU(3) using this approach is a
smoothly expanding and contracting sphere. You can see both U(1) −
the circle − and SU(2) − the biased sphere − in the smooth sphere.

I also think about the group Diff(M), which smoothly alters the sizes
of things depending on where one happens to be in a manifold. There
is an animation of that too.

doug

The groups of the standard model and gravity (the 5 videos below):
http://www.youtube.com/watch?v=ExNPiMcVXww
The group U(1) http://www.youtube.com/watch?v=KZeULLKHE7w
The group SU(2) http://www.youtube.com/watch?v=OMnNyyZruuE
The group U(1)xSU(2) http://www.youtube.com/watch?v=Jbdj3Xd_nmI
The group SU(3) http://www.youtube.com/watch?v=8T_aNL8LvCs
The group Diff(M)xSU(3) http://www.youtube.com/watch?v=pYiEV8yEZYA

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