Re: Gravity on a torus

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On 2007−10−28, pirillo <ultraman2002@xxxxxxxxxxx> wrote:

> This discussion is funny to me because apparently all the
> string theory guys have solved this already−−−since they
> speak of the description of gravity in an alternative world
> say 2 space dimensions,
> where one dimension is compactified and they claim to know
> what the gravitational potential would be in such a case.

Sure, string theorists sometimes include gravity on the 2D worldsheets of strings, but most of the time it doesn't have much effect. The way that gravity is defined in other than 4 dimensions, nowadays, is using the Einstein−Hilbert action of general relativity (which is the integral of the Ricci curvature scalar over space−time). In 2 and 3 dimensions, this version of gravity behaves very differently from what you get by reducing Newtonian gravity to 2 or 3 dimensions (which is described by the 1 or 2 dimensional Poisson equation for the gravitational potential).

In 2 and 3 dimensions, general relativity is in a sense trivial. Einstein's field equations are R = T, where R is the Ricci curvature tensor and T is the energy−momentum tensor. In 2 and 3 dimension, the full Riemann curvature tensor can be altebraically related to the Ricci tensor R. So, in the absence of matter, R = 0 implies the Riemann curvature is also zero, which in turn implies that the metric is flat. A flat metric implies no acceleration due to gravity. This is very different from lower dimensional Newtonian gravity. Solving the Poisson equation in in 1 or 2 dimensions still produces a non−trivial gravitational potential.

For higher dimensional extensions of gravity, Newtonian equations still work in the weak field limit. However, if some of these higher dimensions are compact, then the shape behavior of the gravitational potential as a function of the compactified coordinates strongly depends on what kind of sources are used for matter. I'm not sure if point sources or other kinds of sources are used in those cases. It is possible that some matter distributions are not allowed, because there will exist no solution to Einstein's equations compatible with them.

One would have to look at this issue in more detail.

Hope this helps.

Igor

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