A small numerical error of A. Einstein

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I seem to have discovered a small numerical error in Einstein's "Quantentheorie des einatomigen idealen Gases. Zweite Abhandlung", Sitzungsberichte der Preussischen Akademie der Wissenschaften, Physikalisch−Mathematische Klasse, 1925, no. 1, pp. 3−14. The error occurs near the bottom of page 13. In Einstein's manuscript, available online, the error occurs at the top of page 15; see <http://www.lorentz.leidenuniv.nl/history/Einstein_archive/Einstein_1925_manuscript/Pages/Einstein_1925_15a.html>

The error is, I suppose, quite insignificant. Nonetheless, I'm curious to know if it has been discovered previously.

For me (a mathematician), the tale started a few days ago in a newsgroup devoted to the computer algebra system Mathematica. (If interested in that thread, "Polylog equations", see <http://groups.google.com/group/comp.soft−sys.math.mathematica/browse_frm/thread/a98f725137c3eabb>.) To explain things succinctly, with Li denoting polylogarithm, given y = Li_{3/2}(lambda) and z = Li_{5/2}(lambda), we wish to obtain z as a function of y. Presumably this cannot be done in closed form using known functions, and so Einstein just gave the first few terms of the Maclaurin series

z = y − 0.1768 y^2 − 0.0034 y^3 − 0.0005 y^4

as an approximation. But the latter two coefficients, if given to four decimal places, should have been −0.0033 and −0.0001 instead.

Using Mathematica, it's very easy to get the first few terms of the Maclaurin series precisely, and then to approximate the coefficients:

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In[8]:= Normal[Simplify[
PolyLog[5/2, InverseSeries[Series[PolyLog[3/2, x], {x, 0, 6}], y]]]]

Out[8]= y − y^2/(4*Sqrt[2]) + (1/8 − 2/(9*Sqrt[3]))*y^3 +
((−18 − 15*Sqrt[2] + 16*Sqrt[6])*y^4)/192 +
(317/1728 + 1/(4*Sqrt[2]) − 1/(2*Sqrt[3]) − 4/(25*Sqrt[5]))*y^5 +
((−9450 − 8435*Sqrt[2] + 2400*Sqrt[3] + 4800*Sqrt[6] + 1728*Sqrt[10])*y^6)/34560

In[9]:= N[%]
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Out[9]= y − 0.176777 y^2 − 0.00330006 y^3 − 0.000111289 y^4
− 3.5405*10^−6 y^5 − 8.38635*10^−8 y^6

Three other little comments on the last page of the published article:

1. The largest value of y considered should be zeta(3/2) = 2.612375...,
rather than Einstein's 2.615 .

2. In equation (18c), instead of N, we should have N^4. (The manuscript is correct in this regard, but the exponent is hard to read.)

3. In the final equation, (22d), Einstein gives an approximate coefficient, −0.186, representing the slope of a linear approximation. I presume that Einstein intended to calculate that slope using the endpoints of the graph of F(y). If that presumption is correct, then we can easily obtain the desired slope precisely in terms of the Riemann zeta function:

(zeta(5/2)/zeta(3/2) − 1)/zeta(3/2)

which is −0.186224... That agrees with Einstein's stated value of −0.186, of course. But, considering his previous numerical errors, I can't understand how he got −0.186 ! As best I can tell, he should have gotten −0.189 instead.

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