Re: Local vs Global Constants of Motion

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- *From*: dan@xxxxxxxxxxx
- *Date*: Sat, 12 Apr 2008 13:30:48 +0000 (UTC)

On Apr 10, 6:06=A0pm, Igor Khavkine <igor...@xxxxxxxxx> wrote:

> On Apr 8, 2:22 pm, d...@xxxxxxxxxxx wrote:
>
> Physically, the phase doesn't change when you add 2pi to it. Therefore, we must identify phi with phi+2pi. Topologically, the phase variable lives on a circle.

Yes, that's the root of the problem! What's interesting to me about this example is that you *can* cover all of phase space with the (q,p) chart, and yet there is still a local constant of motion that can't be extended globally.

Conversely, Thirring gives an example of a 2−D manifold that *can't* be covered with a single chart, and yet, for at least some vector fields,*does* have a global constant of motion: manifold is a torus, coords (phi1, phi2) with vector field (w1, w2). If w1 and w2 are rationally related, then sin(w2 phi1 − w1 phi2) is globally defined.

I know it's a basic fact about vector fields on manifolds that they can be "straightened" locally, but not always globally. This somehow freaks me out, but I'm not sure why; I'm very comfortable with the fact that there are tons of coordinates systems on, for example, R^2, that aren't defined globally. It should probably freak me out more that vector fields can be straightened at all than that the straightening isn't always global!

Thanks for the response,

Dan

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