Re: gauge theory (alternative descriptions)

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Thanks for the responses Alexey and Stephen,

The sort of setting I'm after ideally doesn't get too intertwined
with other subjects. I'd rather not deal with groups, bundles,
cohomology, observables,....if that can be avoided.

I'll try to work out "u(1)" gauge algebra (electomagnetism) as
an example; although keep in mind that there are gaps in the
definitions that I'm not sure how to fill. Let's call this
algebra X. I don't know if X should be over a simple field
(R or C) or something more complicated (functions over R,...).
X can be "generated" by 8 elements : X=<d1,d2,d3,d4,a1,a2,a3,a4>
Here "generated" is along the lines of universal enveloping algebra
(polynomials in the 8 generators). You can think of the di's as
partial derivatives, and ai's as related to electromagnetic 4−
potential;
but that's only for motivation...strictly speaking these are just
generators that obey certain multiplication rule. What are these
rules? let the algebra multiplication be g1g2, then define a second
multiplication (commutation) on the algebra g1*g2=g1g2−g2g1. Then
I think all the rules you need are :

di*dj=0; ai*aj=0; di*aj=bij;

the first just says derivatives commute; the second is because this
is an abelian gauge; the last one is a definition of 16 elements of
the algebra; related to partials of 4−potentials. Higher derivatives
can also be treated as definitions of other elements. I think that's
it.

What can you do with this. Let's try to derive Maxwell's equations!
First define "covariant derivative" :

Di=di+ai;

these are just 4 elements in the algebra

Next define the "field strength" :

Fij=Di*Dj;

These are just 16 elements in the algebra. Because g1*g2=−g2*g1,
only 6 of these are significant; these can be identified with
the electric and magnetic field (E1,E2,E2,B1,B2,B3).

Take the identity g1*(g2*g3)+g2*(g3*g1)+g3*(g1*g2)=0 and
substitute Di's in it and with the right identification
of Fij with E and B you get \Del.E=0 and \Del x E = \partial B /
\partial t
Note that these are 4 equations relating three components at a time.

Anyway u(1) gauge is probably the simplest example. It would be
good to find a reference where a non−abelian gauge is treated
along the lines of the above example...

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