Re: neophyte GR question

Source: http://sci.tech−archive.net/Archive/sci.physics.research/2008−07/msg00180.html

- *From*: "Jonathan Thornburg [remove −animal to reply]" <J.Thornburg@xxxxxxxxxxxxxxxxx>

- *Date*: Tue, 29 Jul 2008 21:07:12 +0000 (UTC)

leithaus <lgreg.meredith@xxxxxxxxx> wrote:

> Something about the Hawking−Ellis presentation of GR has long bothered
> me. [[...]]
> what bugs me is that charts don't come for free. From meter rods
> to GPS to atomic clocks, no physically useful coordinate machinery
> comes without a footprint in the stress−energy tensor. Moreover, i
> don't see how this is bootstrapped away in the development of the
> equations. Am i missing something, or is this a bug?

I would say it's not a bug, it's a mathematical model (idealization and abstraction) of reality. Within the formal system of mathematics, we can use coordinates (and other such concepts) to reason about the properties of solutions of the Einstein equations.

For example, we can show that the Schwarzschild metric (written in whatever coordinates you like) is a solution of the vacuum Einstein equations, we can determine the physical interpretation of (say) an areal radial cooridinate, and we can integrate the geodesic equation to find the coordinate trajectories of geodesics.

So far this is all perfectly good *mathematics*, but it doesn't yet have anything to do with *physics*. If we want it to be physics, then we need to make a connection to the "real world". This means we need to consider realizations of our various abstractions/idealizations. For example, to investigate the behavior of the GPS system we might idealize (approximate) spacetime near the Earth as part of Schwarzschild spacetime, i.e. we might approximate the universe as containing a spherically symmetric Earth and nothing else.

In this case the particular approximation you mentioned in your question (ignoring the stress−energy tensor of the coordinate−measuring apparatus) is *very* good: the GPS satellites have a mass that's 21 or so orders of magnitude smaller than that of the Earth, so their stress−energy perturbations to the Einstein equations in the vicinity of the Earth are going to be on the order of one part in 10^21.

Of course, there are lots of other approximations here, e.g. the

Earth isn't really spherically symmetric, there are gravitational tidal forces from the Moon, Sun, and other massive bodies in the universe, and the (cosmological) structure of the universe may well differ from the "asymptotically flat" which is implicit in the Schwarzschild solution.

Much of the "art of physics" consists of correctly judging which approximations are good ones, and which are dubious. For example, the Schwarzschild−spacetime approximation turns out to be excellent for calculating (say) the effects of gravitational redshift on the GPS signals. But if we want to investigate the GPS satellites' orbits around the Earth, then we'd better take into account the non−sphericity of the Earth and the gravitational tidal forces of the Moon and Sun.

ciao,

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−− "Jonathan Thornburg [remove −animal to reply]" <J.Thornburg@xxxxxxxxxxxxxxxxx>
t <= 31.Aug.2008: School of Mathematics, U of Southampton, England
t > 1.Sep.2008: Dept of Astronomy, Indiana University, Bloomington, USA
"Washing one's hands of the conflict between the powerful and the powerless means to side with the powerful, not to be neutral."
−− quote by Freire / poster by Oxfam

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