Re: Fun with the Dirac Delta

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From: zigoteau (*zigoteau_at_ausi.com*)
Date: 06/19/04

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Date: 19 Jun 2004 01:55:51 −0700
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spamspamspam3@netzero.com (Edward Green) wrote in message
news:<eca320d0.0406181515.3bd21f3e@posting.google.com>...

Hi, Edward,

I welcome the improved tone of your posting. This is Usenet, and I can take it, but I do prefer constructive exchanges.

> <On the correct boundary condition for a particle obeying the
> time−dependent Schroedinger equation, and completely localized at t=0>
>
> > You said in one post of this thread that it is not psi(x,0) that
> > should be equal to delta(x), but mod(psi(x,0))^2. That is
*> > unfortunately *>meaningless<*. The delta function was introduced by Dirac*
> > to fill a perceived need, but it is not a nice function. It is defined
> > as a limit, and in quite a few cases expressions involving it converge
> > when you take the limit in the prescribed way. However not all
> > expressions involving the delta function converge correctly. They are
> > not meaningful. Whenever you deal with the delta function, you must
> > keep these problems in mind.
>
> <emphasis added>
>
> Fine words, but I think destined to impress nobody who's opinion is
> worth a tinker's damn.
>
> Let's look at this in detail:
>
> We have two functions connected in some way with the problem. For
> neutrality, lets call them g and h: "g" is understood to be psi, "h"
> |psi|^2. _You_ assert the correct interpretation of the boundary
> condition is to let g be a Dirac delta function a t=0,

Not, "correct". It is the only one that meaningful math comes out of.

You appear not to have appreciated my mathematical derivation of 9 June, which I also include as an appendix to this post. It answers all the objections you have ever come up with (to the mathematics, anyway). I show that if you choose a series of starting conditions which converge on the situation you want to discuss, i.e. |psi(x,0)|^2 = delta(x), then the solution for positive t converges to zero for all x. This is what I meant by my statement that the assumption |psi(x,0)|^2 = delta(x) was meaningless.

> I suggest that
> we want to let h have this property. You respond not by saying my
> suggestion is wrong for such and such a reason, or will not work, but
> that it is _meaningless_. But you produce no evidence for this
> assertion other than a general pandemonium about the subtle issues
> surrounding the blessed delta functions, which I am supposed to be on
> the wrong side of, because ... well, because you say so.

Could I invite you once again to look at the mathematical derivation of the solution which follows from your suggestion. May I reiterate that the delta function is not a function in the normal sense of the word, but is a notation for the limit of a series of functions. If that limit does not exist, then there is no meaning which can be attached to the notation.

The assumption that |psi|^2 is initially the delta function leads to a nonsensical result. Therefore, I stand my my statement. In a well−accepted sense of the word 'meaningless' (see below), your suggestion was meaningless.

I also stand by my statement that Gaussians do not occur naturally in quantum mechanical problems of ballistic transport in the way in which you originally suggested. They occur naturally in statistics, and in problems involving diffusion, but not here. I was hoping that you might learn from my insight, and I was a lot less bombastic than you have been since.

I hope that instead of the tirades of invective which you have produced up until now, you will start responding with specific mathematical objections to steps in my derivation, if you can find any.

<snip>

As a further basis for reasoned discussion, could I point out to you the following definitions:

bullshit[1] noun [uncountable] informal
a rude word meaning something that is stupid and completely untrue: Forget all that bullshit and listen to me! | a load of bullshit: Your so−called plan is a load of bullshit. (Longman's web dictionary, http://www.longman.com/dictionaries/webdictionary.html?lwdEdit=bullshit&submit=OK)

meaningless adjective

1 something that is meaningless has no purpose or importance and does not seem worth doing or having; futile: a meaningless existence

2 not having a meaning that you can understand or explain: To me the marks on the page were just meaningless symbols.
— meaninglessness noun [uncountable]
(Longman's web dictionary,
http://www.longman.com/dictionaries/webdictionary.html?lwdEdit=meaningless&submit=OK)

bal·lis·tic adj.
1. a. Of or relating to the study of the dynamics of projectiles.
b. Of or relating to the study of the internal action of firearms.
2. Of or relating to projectiles, their motion, or their effects.
Idiom:
go ballistic Slang
To become very angry or irrational.
(American heritage dictionary,
http://www.yourdictionary.com/ahd/b/b0044400.html)

A Gaussian function is a function of the form: f(x) =
a*exp[−(x−b)^2/c^2] for some real constants a > 0, b, and c.
(Wikipedia, http://en.wikipedia.org/wiki/Gaussian_function)

Cheers,

Zigoteau.
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APPENDIX

psi(x,0,a) = pi^−0.25/sqrt(a)*exp[−(x/a)^2/2]

I have chosen this so that the integral of |psi|^2 is always unity, and as a−>0 it shrinks down to the point x=0

The Fourier transform of this for wavevector k is

pi^0.25*sqrt(2*a)*exp[−(2*pi*a)^2*k^2/2]

Now add in the phase factor appropriate for this wavevector after a passage oftime t. The resulting wavefunction is:

psi(x,t,a) = integral_−infty^+infty[exp{(2*pi*a*k)^2/2+2*pi*i*(h*t*k^2/2/m+k*x)}*dk]

The expression in the exponent is equal to

phi = A*k^2/2 + B*k

where A = (2*pi*a)^2 + 2*pi*i*h*t/m
and B = 2*pi*i*x

The standard method is to add and subtract the term B^2/A/2 required to complete the square, exactly as in the solution of a quadratic equation. B^2/A/2 is independent of k, so the subtracted term, brought out into a separate exponential factor, just multiplies the integrand by a constant. The integrand, exp the remaining perfect square, is readily converted to the form exp[−A*k^2/2] and its integral equals 1/sqrt(2*pi*A). The end result is unfortunately just a little bit messy:

psi(x,t,a) = pi^0.25*sqrt(2*a)*/sqrt(2*pi)/sqrt((2*pi*a)^2 +
2*pi*i*h*t/m) *exp[−x ^2/(a^2 + i*h*t/m/2/pi) /2]

As a−>0 for t/=0, this converges to zero for all x.