Re: function of state vs exact differentials

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From: Alfred Einstead (*whopkins_at_csd.uwm.edu*)
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"Anja" <anja@no.spam.com> wrote:
> if X is a function of state then is dX an exact differential?
> Is it actually a biimplication?

That's not the correct question. An exact differential form W is always of the form dX for some function, by definition of "exact".

The question you're trying to ask is whether a *closed* differential form W is always exact, W = dX. W is closed if dW = 0. For 1−forms, this boils down to the question of whether and when (for instance) W = A dx + B dy with dA/dy − dB/dx = 0 implies that A = dX/dx and B = dX/dy. For 2−forms, for instance, with W = A dy^dz + B dz^dx + C dx^dy the question is whether and when dA/dx + dB/dy + dC/dz = 0 implies that A = dX/dx, B = dX/dy and C = dX/dz.

If the variables (x,y) range over some domain S in which the following is true:

> every path P from any points (x0,y0) to (x1,y1)
> in S can be deformed continuously to any other
> path P' from (x0,y0) to (x1,y1)

then it is true that dA/dy − dB/dx = 0 implies A = dX/dx,B=dX/dy. A similar condition holds for the second example cited for the 2−form W = A dy^dz + B dz^dx + C dx^dy.

In the case you're interested in, if the system's state is given by the coordinates (q1,...,qn,p1,...,pn) for some n > 0, then a 1−form W is given by

W = Q1 d(q1) + ... + Qn d(qn) + P1 d(p1) + ... + Pn d(pn).

The domain S over which (q1,...,qn,p1,...,pn) should have the above−mentioned path property. Then it will be true that a closed differential form W

dW = 0, that is:
d(Qi)/d(qj) = d(Qj)/d(qi); i,j = 1,...,n
d(Qi)/d(pj) = d(Pj)/d(qi); i,j = 1,...,n
d(Pi)/d(pj) = d(Pj)/d(pi); i,j = 1,...,n

implies W is exact, W = dX, or

$Qi = dX/d(qi)$, $Pi = dX/d(pi)$; $i = 1,...,n$.