Re: Epistemology 201: The Science of Science

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"Jason" <jasonstevensNOSPAM@free.net.nz> writes:

>> >Maths is an extension of FOPC, like PA.

>> Not really. Mathematics is much older than FOPC, so it doesn't make
>> sense to say it is an extension of FOPC.

>Okay, this is really strange to me because this is so not what I've come to
>understand mathematics as. These days, in mathematical reasoning, logical
>arguments are used to deduce consequences (theorems) of the assumptions of maths
>(axioms). Most of maths is built from sets, so the basic assumptions of maths
>are the axioms of set theory, in particular ZFC set theory. [Chapter Zero −
>Fundamental Notions of Abstract Mathematics, Carol Schumacher]

You appear to be thinking of mathematics as a branch of logic. But
it is the other way around −− logic is a branch of mathematics.

>You are suggesting that maths is not this formal system, so I am lead to assume
>that you have some sort of prior understanding of what is mathematically legal
>and illegal, like most people.

"Mathematically legal" is a matter of logic. But mathematics is not
just logic.

> But is this type of reasoning informal or have
>we our own set of assumptions, much like axioms, that enable us to perform
>mathematical inference. When there is a disagreement, where do we turn? From
>my understanding it is this formalised system of mathematics, which took root
>with Whitehead and Russell in the principia mathematica. Hence its FOPC roots.

Keep in mind that mathematics is far older than Russell & Whitehead's
Principia.

>I can accept that the axioms are not often invoked in the heat of proofs, but
>then neither is the road−code when we are driving. Axioms as such don't need to
>be the way to go either. The more intuitive way to go are to use rules of
>inference, which are equivalent and perhaps closer to the story about how we

>'do' maths.

>Out of interest, if maths is not this formal system then how can abstract
>mathematics take place? For example, how can the continuum hypothesis be
>(dis)proven, or proved not to be provable?

The continuum hypothesis is a side show. For sure, some people find
it an interesting side show. But it is still only a side show.

As a first approximation, think of mathematics as the study of
pattern or regularity. When mathematicians come across an
interesting pattern or regularity, they will attempt to characterize
that regularity. This characterization might be in the form of a set
of rules. Then they will use these rules as axioms, and investigate
their consequences.

Axiom systems are not the starting point of mathematics. They are
often one of the products of mathematics.

>> >and assumed, as far as I am aware.

>> Again, not really. Mathematicians often try to make do with minimal
>> axioms.

>Which ones? The choice is critical to what is provable and what isn't.

It depends on what is being studied. For many purposes, you can get
away with the axioms of a field. For other purposes, you will also
need an axiomatization of topology.

>> If you happen to be making a vague reference to the Banach−Tarski
>> paradox, then you have it wrong. Banach−Tarski does depend on the
>> axiom of choice.

>I went to a seminar on this last year and I thought the dude said the problem
>went away with invoking the axiom of choice. But now having read some more I
>realise I misunderstood. Okay, bad example.

>How about another then. It has been proven that in ZFC set theory, the formal
>system of mathematics (I honestly can't see why you flatly refuse that there is
>such a system), the continuum hypotheses is can neither be proven or disproven.
>So it could be asserted true or false with a new axiom and there would be two
>overlapping but distinct mathematical universes to choose from.

Right. Should I add a yawn?

>If ZFC is assumed as the foundations of maths, it has been shown by Chaitin that
>there are infinite arithmetic truths that cannot be proven in ZFC. Where does
>maths as not−a−formal−system fit into this?

Mathematics can manage pretty well without a foundation. Foundations come and go, but most of mathematics continues unperturbed.

>> There you go again. You talk about "the formal system of maths", but there
>> is no such formal system. Then you suggest that we should instead
>> study some other formal system. It is gibberish.

>Is ZFC set theory a small and inconsequential part of mathematics?

I'm not sure I would call it small, and I certainly would not say that it is inconsequential. Nevertheless, it is only part of mathematics. Do keep in mind that calculus is several hundred years older than ZFC.

> I suppose
>you don't really get into it unless you study number theory, mathematical logic
>and stuff, but it was my understanding that this system what the foundation of
>modern maths.

It isn't the foundation of mathematics.

There is a sub−discipline of mathematics that studies foundations. And within that subdiscipline, people attempt to find out how much of mathematics can be built on ZFC as a foundation. That's a very interesting exercise. But it doesn't follow that mathematics really does depend on ZFC as a foundation.

Here is an analogy. It can be shown that all logic operations are derivable from "not and" (what the NAND gate does). But it does not follow that all actual logic chips are built out of NAND gates.