Quantum Gravity 218.3: Riccati Differential Equation vs Generalized Exponential Function

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From Osher Doctorow

The Generalized Exponential Function (GEF for short) has the formal form for one argument x:

1) GEF(x) = sum ki x^i (sum from i = 0 to infinity)

where ki is either +/− 1/i! or 0 for i = 0, 1, 2, ..., and the ki are allowed to be phase dependent or piecewise constant.

It generates exp(x), exp(−x), cosh(x), sinh(x), sin(x), cos(x), and I am allowing x to be replaced by cx, x +/− c, and so on for constant c.

The Riccati Differential Equation is:

2) dy/dt = A(t) + B(t)y + C(t)y^2

With t = x and the coefficients A(t), B(t), C(t) themselves either arbitrary (or specified) or phase constants including 0, dy/dt of equation (2) generates exp(kx), exp(−kx), cosh(kx), sinh(kx), and if we allow complex solutions exp(ikx) for k real, it generates sin(kx), cos(kx). It's typical to choose A(t), B(t), C(t) as either constant (or in my case phase constants) or simple functions of time t like simple polynomials or power functions (including negative powers) to simplify things but also in my view because the restriction of (2) to the second power or less of y makes the restriction of the coefficients A(t), B(t), C(t) to the second power or less of t or similar things more symmetric and more plausible. Once in a while there may be a reason to choose more complicated functions for the coefficients from the physics, but putting no restrictions on them would seem to defeat the idea of simplification and brevity.

It is rather interesting arguably that the GEF and the "reasonable" or "second degree or less coefficient" Riccati Differential Equation yield similar solutions, although the Riccati DE seems to yield more including rational functions of exponentials as in the Logistic

Quantum Gravity 218.3: Riccati Differential Equation vs Generalized Exponential Function Differential Equation subcase.

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