Re: The opposing rockets and the box

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- *From*: Owen Jacobson <angrybaldguy@xxxxxxxxx>

- *Date*: Tue, 22 Jul 2008 23:08:07 −0700 (PDT)

On Jul 22, 11:56 pm, "Spaceman" <space...@xxxxxxxxxxxxxxxxxxxxxxxxxx>
wrote:

> Greg Neill wrote:
>
>> Amazing. James has discovered an infinite energy source
>> and perpetual motion! How could it be that 2000 years of
>> tinkering with colliding masses has not produced one other
>> person who has noticed this!
>
> See,
> Still no simple proof and just arrogant asshole bullshit
> twisting instead of a simple answer for the KE I asked
> him to prove.
>
>> But it wouldn't have a larger KE. In fact its KE would
>> be much lower than that of the original projectile.
>
> So why didn't you show such math Greg?
> It would have been very simple to do.
> But instead you sat on your highhorse
> blabbing about momentum blah blah blah.
> Sad Greg.
> I figured you would prove yourself an asshole
> and you did it very well.
> You let me go on and on to make yourself
> feel even more smarter than ever.
> To bad you were actually being tested and
> failed.
> You are an ass.
> I was trying to see if you were not the big ass
> I thought you were, but you proved you are.
> Thanks for the proof ASS!
>
> BTW asshole.
> I did the numbers before I posted this.

> And I knew the whole time the KE would
> not be larger for real because there is no
> way a collision will "gain" KE.
> but of course. you showed how much an ass you are.
> Thanks again.. ASS!
> LOL

I actually prepared this for your ball−in−a−box example.

Let's see if we can come up with the result predicted by conservation of momentum without actually using it as a law. First, some definitions, per high−school physics (no relativity here, only Sir Isaac Newton and friends).

Fundamentals:

Time is a separation between two events; its units are seconds (s).
Position is a vector describing where something is in relation to a reference point; its units are metres (m).
Mass is a quantity of matter; its units are kg (for convenience).

Then:

Velocity is a change in position over time; its units are m / s.
Acceleration is a change in velocity over time; its units are m / s / s.

Momentum is a property of mass in motion, described by the equation p = m * v. Its units are kg * m / s.
Force is an acceleration of a mass, or a change in momentum over time. It is described by the equation F = ma. Its units are Newtons, or, equivalently, kg * m / s / s.

Newton's laws of motion, informally, in terms of the above definitions:

1. An object will stay at rest or continue at a constant velocity unless acted upon by a force.

2. F = ma: the force on an object is equal to the mass of the object multiplied by its acceleration.

3. Whenever an object A exerts a force on another object B, B simultaneously exerts a force on A with the same magnitude in the opposite direction.

Finally, an example:

There is a box, 1 000 m long with a 100 g weight centered at the front. The box weighs 1 kg. Both the box and the weight are at rest (velocity = 0). The entire system is under no external forces and is in total vacuum; for the following, assume the motion of the weight within the box and the motion of the box and the weight within their environment is frictionless.

From this and the definition of momentum above, we can come up with the momentum of the weight:
Pw = Mw * Vw

We know that Mw = 100 g = 0.1 kg and Vw = 0 m / s from the definition, so we can substitute them to give
Pw = 0.1 kg * 0 m / s
Pw = 0 kg * m / s

That is, the weight has zero momentum. Intuitively, this makes sense; the weight is a mass that is not in motion.

Similarly, using Mb = 1 kg and Vb = 0 m / s, we can calculate the momentum of the box:
Pb = Mb * Vb
Pb = 1 kg * 0 m / s
Pb = 0 kg * m / s

This is consistent with the result for the weight, as the box is also at rest.

Finally, we can compute the initial momentum of the whole system as the sum of the momenta of the parts
Pinitial = Pw + Pb
Pinitial = 0 kg * m / s + 0 kg * m / s
Pinitial = 0 kg * m / s

At time t = 0, the box exerts a force of 10 N on the weight for one second, causing the weight to accelerate towards the back of the box. The acceleration of the weight is described by
Fw = Mw * Aw

We know Fw = −10 N (arbitrarily chosing the negative side of the coordinate system to represent force, motion, and positions towards the back of the box) and Mw = 0.1 kg from the problem; we can substitute these values into the above to get
−10 N = 0.1 kg * Aw

which can be rearranged to give the acceleration of the weight
Aw = −10 N / 0.1 kg = (−10 kg * m / s / s) / 0.1 kg
Aw = −100 m / s / s

By the third law above, the weight exerts an equal 10 N force on the box, causing the box to accelerate forward, described by
Fb = Mb * Ab

We know that Fb = −Fw = 10 N and Mb = 1 kg; we can substitute these in to get
10 N = 1 kg * Ab
Ab = 10 N / 1 kg = (10 kg * m / s / s) / 1 kg
Ab = 10 m / s / s

After one second, the velocity of the weight towards the back of the box is described by
V'w = Vw + Aw * t

Substituting in the acceleration Aw from above and a time of 1 second from the problem, we get
V'w = 0 m / s + −100 m / s / s * 1 s
V'w = −100 m / s

Similarly, after one second the forward velocity of the box is described by
V'b = Vb + Ab * t

Once again substituting in values, we get
V'b = 0 m / s + 10 m / s / s * 1 s
V'b = 10 m / s

Now we can again calculate the momentum of the weight:
P'w = Mw * V'w
P'w = 0.1 kg * −100 m / s
P'w = −10 kg * m / s

Examined on its own, the weight now has momentum. This makes intuitive sense too −− it's moving, rather quickly. Let's look at the box's momentum
P'b = Mb * V'b
P'b = 1 kg * 10 m / s
P'b = 10 kg * m / s

That's interesting: the box's momentum is the same size, but in the opposite direction. And the whole system's momentum is now
Pmoving = P'b + P'w
Pmoving = 10 kg * m / s + (−10) kg * m / s
Pmoving = 0 kg * m / s

The system as a whole has gained exactly zero momentum! *This* is the phenomenon known as conservation of momentum, and you can apply the same math to decelerating the weight (and the box) when the weight collides and adheres to the back of the box. We'll look at that in a moment.

This might look like a simple exercise in algebra, and on its own it would be. However, Isaac Newton and the multitude of physicists that came before and have come after him experimentally verified all of the relationships given as definitions at the beginning of this post, and the experiments are both well−documented and easy to reproduce for yourself. In fact, these experiments are often part of a high−school−level physics education.

−− intermission −− get a drink −− intermission −−

Still here? Ok. You may have noticed I completely disregarded kinetic energy in the exposition above. There are two reasons for this: one, you don't need it; classical mechanics for linear motion can be completely modeled using force, mass, and momentum; and two, kinetic energy isn't conserved, so it isn't useful for deciding whether the system as a whole will remain in motion. Let's go back to our weight, which is now moving at V'w = −100 m / s (that is, very quickly towards the back of the box).

In classical mechanics, kinetic energy is, like momentum, a property of mass in motion. The equation describing it, however, is different: E = m * v * v / 2. It has units Joules (J), or equivalently Newton−metres (N * m), or equivalently kg * m * m / s / s.

We can calculate the kinetic energy of the weight:
Ew = Mw * V'w * V'w / 2
Ew = (0.1 kg * −100 m / s * −100 m / s) / 2
Ew = (1000 kg * m * m / s / s) / 2
Ew = 500 J

We can also calculate the kinetic energy of the box:
Eb = Mb * V'b * V'b / 2
Eb = (1 kg * 10 m / s * 10 m / s) / 2
Eb = (100 kg * m * m / s / s) / 2
Eb = 50 J

The weight, despite (actually, due to) being lighter than the box, has ended up with the majority of the kinetic energy. In fact, all of that kinetic energy was introduced into the system by accelerating the weight; when the system was initially at rest, both the weight and the box had 0 J of kinetic energy.

Now, let's talk about what happens at the far end of the box. For the sake of simplicity, let's assume the weight flies freely down the box and collides with the end, adhering nearly instantly without deforming. This means the final velocity V''w of the weight and the final velocity V''b of the box must be equal; they're now attached to each other, so they must be moving together. For the same of simplicity, let's assume that this deceleration takes 0.001 s. (The calculus−savvy posters can tell you how far the weight moves with in that time.)

Since the deceleration is a result of a collision between the box and the weight, the weight is applying a force F'b to the box, and the box is applying an equal and opposite force F'w to the weight.

F'w = −F'b

We also know that the acceleration of the weight A'w and box A'b are given by
F'w = Mw * A'w
F'w = 0.1 kg * A'w
and
F'b = Mb * A'b
F'b = 1 kg * A'b

From the third law, we know that F'w = −F'b, so we can eliminate a variable and come up with a relationshp between the accelerations of the box and the weight:
F'w = A'w * 0.1 kg = −1 * A'b * 1 kg = −F'b
A'w * 0.1 kg = −1 * A'b * 1 kg
A'w = −1 kg * A'b / 0.1 kg
A'w = −10 * A'b

We also know that the final velocities of the weight and the box are equal
V"w = V"b

since the weight and the box become attached and move together after the collision, and that
V"w =V'w + A'w * t'
V"w = −100 m / s + A'w * 0.001 s
and
V"b = V'b + A'b * t'
V"b = 10 m / s + A'b * 0.001 s

So we can eliminate a variable:
V" = −100 m / s + A'w * 0.001 s
V" = 10 m / s + A'b * 0.001 s

This gives us a second relationship between the two accelerations:
−100 m / s + A'w * 0.001 s = 10 m / s + A'b * 0.001 s
−110 m / s + A'w * 0.001 s = A'b * 0.001 s
A'w * 0.001 s − A'b * 0.001 s = 110 m / s
(A'w − A'b) * 0.001 s = 110 m / s
A'w − A'b = (110 m / s) / 0.001 s
A'w − A'b = 110 000 m / s / s

We can plug the first relationship into this to get the acceleration of the weight
(−10 * A'b) − A'b = 110 000 m / s / s
−11 * A'b = 110 000 m / s / s
A'b = (−110 000 m / s / s) / 11
A'b = −10 000 m / s / s

Which we can then plug back into the second relationship to determine the acceleration of the box:

A'w − A'b = 110 000 m / s / s
A'w − (−10 000) m / s / s = 110 000 m / s / s
A'w + 10 000 m / s / s = 110 000 m / s / s
A'w = 100 000 m / s / s

Just as a refresher, since that was a fairly large Wall O' Algebra: immediately prior to the collision, the box is moving forward at 10 m / s and the ball is moving backwards at 100 m / s; the collision takes 0.001 s. Now that we have the accelerations of the weight and the box, we can work out their final velocities:
V"w = V'w + A'w * t'
V"w = −100 m / s + 100 000 m / s / s * 0.001 s
V"w = −100 m / s + 100 m / s
V"w = 0 m / s

Surprise! The weight is now at rest again with regards to our original system. Given that we know the weight is attached to the box, the box must also be at rest again, but let's work it out anyways:
V"b = V'b + A'b * t'
V"b = 10 m / s + −10 000 m / s / s * 0.001 s
V"b = 10 m / s − 10 m / s
V"b = 0 m / s

The total momentum of the system is still 0:
P"w = Mw * V"w
P"w = 0.1 kg * 0 m / s
P"w = 0 kg * m / s

P"b = Mb * V"b
P"b = 1 kg * 0 m / s
P"b = 0 kg * m / s

Pfinal = P"w + P"b = 0 kg * m / s

Looks like momentum truly is conserved, even if you treat momentum as an combination of mass and velocity, rather than as a fundamental property of matter in motion. I don't think I need to point out that all of the kinetic energy introduced by accelerating the weight has been removed from the system again; thermodynamics can tell you where it went.

How far the ends and centre of mass of the assembly have moved and how much time has elapsed are left as an exercise to the reader.
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