

Re: Stochastic Taylor series

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- *From:* hrrubin@xxxxxxxxxxxxxxxxxxxxxxxx (Herman Rubin)
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In article <96335dfc-8f88-40be-9eae-d5026f0e7e39@xx>, Peter <pwolynes@xxxxxxxx> wrote:

Hi,

I am wondering if the following type of Taylor series is known .

I am working with the following quantity:
 $z = \text{ArcTan} [(\sin(a) + x)/(\cos(a) + y)]$
where x, y i.i.d. $N(0, s^2)$ where $s < 1$ (we'll say $s \ll 1$ to simplify the problem)

I want to study the statistical properties of z . Now, the distribution of z can be worked out (since the distribution of the ratio of two independent normal rvs is a known one), but in my case, the distribution is too complicated to work with very easily.

As a consequence, I want to do a Taylor series expansion of z wrt x and y (about $x=0$ and $y=0$).
The form for the Taylor series has a nice simple form (esp for the first few orders):
 $z = a + \cos(a) * x - \sin(a) * y - \sin(2a) * (x^2/2) - \sin(2a) * (y^2/2) - \cos(2a) * x * y.$

Since x and y are statistically independent, computing approximations to the mean, variance, etc of z can then be performed rather easily.

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Technically, however, I am wondering how valid this is. x and y will almost surely have a magnitude less than unity, but cases may arise in which the magnitude of x relative to $\cos(a)$ may not be small, and it is not obvious to me that that wouldn't cause problems.

Thanks in advance for any help/insight on whether people perform this type of Taylor series expansions and what conditions are required for them to be well-defined.

Peter

The Taylor series here does not converge for all x and y , as the ArcTan has singularities at $\pm i$. For small s , the approximation does have value, and the moments can be approximated from the series. Large values of either x or y can produce problems. Since ArcTan is bounded for real arguments, these approximations MIGHT be good.

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This address is for information only. I do not claim that these views are those of the Statistics Department or of Purdue University.
Herman Rubin, Department of Statistics, Purdue University
hrubin@xxxxxxxxxxxxxxxxx Phone: (765)494-6054 FAX: (765)494-0558
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