Re: A simple but confusing question

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From: Ian Jermyn (*ianjermyn_at_wanadoo.fr*)
Date: 10/03/04

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Date: Sun, 3 Oct 2004 20:33:48 +0200
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It seems that Lewis Carroll was correct (no surprise). Computing

Pr(w_{n + 1} | W_{n}) = Pr(W_{n + 1}) / Pr(W_{n})

with

Pr(W_{n}) = \sum_{N} \sum_{M < N} (M/N)^{n} Pr(M | N) Pr(N) ,

where 1 <= N <= N_{0}; Pr(M | N) is binomial with p = 1/2; and Pr(N) =
1/N_{0}; gives

Pr(w_{n + 1} | W_{n}) = 1/2

as N_{0} goes to infinity, at least under the slight approximations I made.

Ian.

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Ian Jermyn
ianjermyn@wanadoo.fr
"Ian Jermyn" <ianjermyn@wanadoo.fr> a écrit dans le message de
news:416029a6$0$21343$8fcfb975@news.wanadoo.fr...
> This is only approximately true, for N >> n, where N is the number of
balls
> in the bucket and n the number of draws. Let w_{n} be the proposition 'the
> nth ball is white'.  Let W_{n} be the proposition w_{n} & w_{n − 1} & ...
&
> w_{2} &w_{1}. Assume we know how many balls (N) are in the bucket. Let the
> number of white balls in the bucket be M. The probability of the next ball
> being white after n white balls is given by
>
> Pr(w_{n + 1} | W_{n}, N) = Pr(W_{n + 1} | N) / Pr(W_{n} | N) .
>
> Then
>
> Pr(W_{n} | N) = \sum_{M = 0}^{N} Pr(W_{n} |  M, N) Pr(M | N)
>
> = 2^{−N} N^{−n} \sum_{M = 0}^{N} M^{n} (N choose M) ,
>
> because Pr(M | N) is binomial with p = 1/2 and Pr(W_{n} |  M, N) is simply
> (M/N)^{n}. Thus
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> Pr(w_{n + 1} | W_{n}, N) = N^{-1}[\sum_{M = 0}^{N} M^{n + 1} (N choose M)
/
> \sum_{M = 0}^{N} M^{n} (N choose M)] .
>
> For n >> N, the sums will be dominated by the terms with M = N,
> and we find that
>
> Pr(w_{n + 1} | W_{n}, N) = N^{-1} N = 1 ,
>
> whereas for N >> n the term with M = N/2 will dominate, and we find
>
> Pr(w_{n + 1} | W_{n}, N) = N^{-1} (N/2) = 1/2 .
>
> This is only natural. When we have much more data than balls in the
bucket,
> the data dominates our estimate of M. When the reverse is true, the prior
> dominates. When both N and n are small, no one term will really dominate
and
> the sum must actually be evaluated.
>
> I am still thinking about the case in which we do not know the number of
> balls in the bucket.
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> Ian.
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> Ian Jermyn
> ianjermyn@wanadoo.fr
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>
> "Henry" <se16@btinternet.com> a écrit dans le message de
> news:mnjrl09pi6r05hjdqnchr4j8cvtpqnfgi7@4ax.com...
> > On Sat, 02 Oct 2004 20:30:38 +0300, George Kahrimanis
> > <anakreon@hol.gr> wrote:
> > > I admit that I feel somewhat like Alice in Wonderland
> >
> > Then use Lewis Carroll's prior in one of his "Pillow Problems", which
> > assumes that the balls were originally put in the bucket at random
> > with each a probability 1/2 white and 1/2 black. After 100 whites
> > drawn from the bucket, the probability of the next drawn being white
> > is still 1/2.
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