

Re: A simple but confusing question

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szhao@darkwing.uoregon.edu (Shanyu Zhao) wrote in message
news:<96a39245.0409270026.63fd76c@posting.google.com>...

> *Here is the naive question:*

>

> *There are a large number of balls in a bucket, the white color balls
> occupy p and the black balls $(1-p)$. If p is unknown, when you pick 100
> balls from the bucket, find that all of them are white. Then you pick
> the 101st ball, what is the probability that the ball still a white
> one?*

The answer can be anything from 0 to 1. (See below.) The answer that I
would consider reasonable is 101/102.

> *Is this problem a parameter estimation or hypothesis testing? If we
> use parameter estimation, clearly $p=1$, which means the probability is
> 100%. But this is not true.*

This is not right.

There are a couple of different definitions of probability. The
frequentist definition states: if you pick n balls, and x of them are
white, then probability of a white ball is $p = (x/n)$ **as n goes to
infinity**. People sometimes forget this last part, and it often does
not matter; it matters here. 100 is not equal to infinity, so you
cannot apply the frequentist definition to calculate probability.

This is an estimation problem. p , the probability, is a parameter that
you want to estimate. A model consists of two parts: a likelihood,
which expresses the interaction between the parameters and the data;
and a prior, which expresses your prior beliefs about the parameters.

For this particular problem, the likelihood is $\text{Binomial}(x | n, p)$. If
the prior is $\text{Beta}(a, b)$, then the best estimator of p (under quadratic
loss) is $p_{\text{hat}} = (x+a)/(n+a+b)$.

What should a and b be? In principle, they could be anything. They
express your beliefs about p before you drew any of the balls. That is

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why I said that your estimate of p could be anything between 0 and 1.
If $a = 1$, $b \rightarrow \text{Infinity}$, then $\hat{p} \rightarrow 0$. But is such a prior
reasonable? Probably not.

If you have no prior information about p , then I would say, set $a = b$
 $= 1$. $\text{Beta}(1,1)$ is the $\text{Uniform}(0,1)$. This prior says that, a priori, p
is equally likely to be anything between 0 and 1. Under this prior,
 $\hat{p} = 101/102$.

Some people would say that if you have no prior information, set $a = b$
 $= 0.5$. (This is the Jeffrey's prior.) I would disagree, but in
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