

Re: Distribution of a transformed Gaussian variable.

Source: <http://sci.tech-archive.net/Archive/sci.stat.math/2005-01/0088.html>

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Date: 01/03/05

Date: 3 Jan 2005 12:42:09 -0800

Fredrik Hekland wrote:

- >I'm trying to find the distribution resulting from taking the square
- >root of the absolute value of a Normally distributed variable. More
- >precisely, $p_z(z)$ when
- > $z = \text{sign}(n) * \text{sqrt}(\text{constant} * \text{abs}(n))$, $n \sim N(0, \text{sigma}_n^2)$.
- >Not having any statistical textbooks at my current location (and not
- >being a statistics wizard), I'm unable to calculate the resulting pdf.
- >Is it a complex task to determine this distribution?
- >Simulations indicate that the resulting pdf is close to a Laplace
- >distribution (double exponential). Does this seem reasonable?

The absolute value of a standardized normal variate follows the half-normal distribution, described at <http://mathworld.wolfram.com/Half-NormalDistribution.html>, with distribution $(2/\pi) * \exp(-x^2/\pi)$. I think the square root of a variate following this distribution has a density proportional to $\text{sqrt}(x) * \exp(-x/\pi)$, for $x > 0$, which has the same decay as a Laplace on the right side but not on the left. Please check this before using it.