Re: convergence of probability measures?

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amitgandhi@gmail.com wrote:
>*I have a question about convergence of probability measures that is*
>*entering*
>*some research I am doing. Here is the situation:*
>
>*−Let us say our measurable space is an interval of the real line X =*
>*[a, b], or a rectangle in R^n more generally.*
>
>*−I have a sequence of probability measures \mu^t over X that*
converges
>*weakly to a limiting measure \mu. Each \mu^t is absolutely continuous*
>*with respect to lebegue measure − i.e. they can be described by*
>*densities. I have a belief as to what the limiting measure \mu is:*
\mu
>*is a delta mass over a special state z \in [a, b]. I want to prove*
>*this.*
>
>*−Here is the strategy I am trying to use for proof − and I wanted to*
>*run it by anyone in this group. Instead of working with the sequence*
>*\mu^t directly, I am going to deal with a disturbed sequence of*
>*probability measures \m_{d}^t, where d is a number that represents a*
>*disturbance parameter in the sense that for every t, as d−>0,*
\m_{d}^t
>*converges weakly to \mu^t.*
>
>*−For every fixed d > 0, the sequence (in t) of probability measures*
>*\m_{d}^t converges weakly and strongly to a limiting probability*
>*measure \m_{d}. Further I know that as d −> 0, these limiting*
measures
>*\m_{d} weakly converge to a probability measure \m, which I know is*
the
>*delta mass over z that was of original interest.*
>
>*−From this, does it follow that I can show that \mu = \m, i.e., does*
>*the limit \mu of the original sequence of probability measures \mu^t*
>*equal \m, and hence the delta mass over z?*
>
>*I was trying to see if I have the technical conditions needed to make*

>this result follow. Thanks to anyone in advanced.

Dear Amit,

If you have trouble reading this, just save this as a .tex file and typeset it in TeX.

You are trying to construct what's called a triangular array of measures for which you have convergence along each fixed row, and each fixed column, and from this, you are trying to conclude convergence along the diagonal. You can think of a 2−dim function (with variables s(say $=1/d$) and t with $s \geq 0$, $t \geq 0$ such that the function has a ridge of constant (or oscilating) height along the diagonal, but for each fixed s(or t) value, it converges to zero as $t \to \infty$(or $s \to \infty$).

Of course you can impose conditions on triangular arrays to ensure convergence along the diagonal. You can find such conditions in one of Konrad Knopp's books on infinite series.

Triangular arrays of measures can have similar behaviour, i.e., a ridge or a bump along the diagonal. If you are really interested, conditions for convergence may be found in Billingsley's book "The convergence of probability measures", or Gnedenko and Kolmogorov's classic "Limit distributions for sums of independent random variables". Look for "triangular arrays", or "diagonal method".

But before that, if you have already guessed the limiting measure μ, you might like to try to use the direct definition of the convergence of measures, i.e., $\mu_{n} \Rightarrow \mu$ iff for every continuous bounded function f,

\[
\int f(x) d \mu_{n}(x) \to \int f(x) d \mu(x)
\]

Best,

Zaeem Burq,

Melbourne