

Re: Anyone found an Elementary Bayesian Book yet?

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David Jones wrote:

No.  $\text{Beta}(r+1+\alpha, n-r+1+\beta)$  as a posterior density would contain  $p$  to the power of  $(r+\alpha)$ , not  $(r+\alpha-1)$  as required.  $\text{Beta}(\alpha+r, \beta+n-r)$  is the answer given in cookbooks.

It's my turn to say No. And this is going to be the last round for me.

The cookbook says  $n'' = n + n'$  and  $r'' = r + r'$

where ALL the  $n$ 's and  $r$ 's are just powers, in  $p^r (1-p)^{(n-r)}$ , the likelihood form, without making an index shift to the  $\alpha$ 's and  $\beta$ 's. I did that to make the point about Beta being the conjugate prior, and I regret doing so now, because it introduced the confusion of mixing one parametrization with a different one.

So, I'll go back the original book from which Russell Martin cited:

RM> Chapter 3 treats Bernoulli  
RM> processes, and beta distributions as conjugate priors.  
RM> Interestingly (if I'm reading it correctly) he suggests using  
RM> Chapter 3 treats Bernoulli  
RM> processes, and beta distributions as conjugate priors.  
RM> Interestingly (if I'm reading it correctly) he suggests using  
RM>  $r=n=0$  as "vague" prior parameters. He acknowledges  
RM> that this gives a prior beta density that is undefined, but  
RM> writes, "Nevertheless, if we ignore this deficiency and  
RM> apply Eq. (3.5) using  $r'=n'=0$  as prior parameters, then  
RM> the posterior parameters become  $r''=r$  and  $n''=n$ .

Russell never showed Eq. (3.5), but from the use of  $r'=n'=0$  for the prior parameters, I assume the author was using the "standard cookbook" form as I explained:

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RF> He even had the primes according to the usual cookbook  
RF> conventions. The  $r$  and  $n$  denote the SAMPLE  $r$  and  $n$ ,  
RF> those in the likelihood function.  $r'$  and  $n'$  denote the  
RF> parameters in the prior distribution  $\text{Beta}(r',n')$  rather than  
RF> alpha and beta. That's because then you have the  
RF> "no brainer" of using the Beta as the conjugate prior,  
RF> because the posterior is given by (double primes)

RF>  $r'' = r + r'$  and  $n'' = n + n'$ .

Russell can check me on that. I was inferring from Russell's cited paragraph that the line above is the Eq. (3.5) with my explanation of  $\text{Beta}(r',n')$  above.

Then Russell's  $B(0,0)$  is the  $r'=n'=0$  of  $1/(p(1-p))$  improper prior. The uniform prior is  $B(1,1)$ , a proper prior, in that notion.

The  $r$  and  $n$  remain the sample data and likelihood  $r$  number of successs out of  $n$  trial.

So, the improper prior, denoted by  $B(0,0) = 1/(p(1-p))$ , is the one that yields the original likelihood function as the identical posterior distribution.

Now we get back to David Jones.

and that's why  $B(0,0)$  will give the likelihood function for the posterior.

No  $\text{Beta}(1,1)$  (uniform) gives (a scaled version of) the likelihood function for the posterior. As noted by others, this doesn't necessarily mean that the uniform can be counted as non-informative ... one of my books says that even Bayes had doubts about making such a claim.

What do you mean by "(a scaled version of) the likelihood"?

$B(0,0)$  gives back the original likelihood function for the posterior.

$B(1,1)$  gives a function that bumps the powers of  $p$  and  $(1-p)$  in the original likelihood by 1, to yield a posterior distribution DIFFERENT from the original likelihood.

Another check is to see what happens when there is no data from the experiment, so that the posterior is the same as the prior. "No

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experiment" would be equivalent to  $n=r=0$

The fly in the ointment is that with no data, you DON'T have a likelihood function and you don't have a posterior density.

But now, reverting back to Russell Martin's book and the convention used THERE, your check works. Because then  $n'' = n'$  and  $r'' = r'$  if you take the data  $n=r=0$ .

David Jones

I think David Jones's book and Russell Martin's book should now agree with the same recipe in the cookbooks from which I've taught:

RF>  $r'' = r + r'$  and  $n'' = n + n'$ .

If not, then I'll just stand by the present version as the version that is consistent in ALL the books I've used, which give

$r'' = r + 1$  and  $n'' = n + 1$

for the uniform prior,  $U(0,1)$ , denoted as  $B(1,1)$  in these books.

— Reef Fish Bob.

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