

## Re: $X+Y \mid -w \leq X-Y \leq w$

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On 29 Jul., 17:19, "Stephen J. Herschkorn" <sjhersc...@xxxxxxxxxxxxxxxx> wrote:

In sci.stat.math, bokdan hermicz wrote:

if  $X$  and  $Y$  are Gaussian and independent and  $w = \text{Inf } P(X+Y \mid -w \leq X-Y \leq w)$  is equal to the unconditional probability  $P(X+Y)$ . This implies that  $Z=X+Y$  follows a Gaussian with mean  $E(X)+E(Y)$  and  $\text{Var}(Z)=\text{Var}(X)+\text{Var}(Y)$ .

I do not understand what you mean by "This implies that." The fact that has normal distribution with parameters  $E(X) + E(Y)$  and  $\text{Var}(X) + \text{Var}(Y)$  has nothing to do with your conditioning event.

If  $X$  and  $Y$  follow a uniform distribution with support  $(0,1)$  then  $Z=X+Y$  (for  $X$  and  $Y$  independent) follows a triangular distribution. But the density of  $P(Z \mid |X-Y| \leq w)$  has a trapezoidal shape with a flat part in the middle, if the condition really binds ( $w < 1$ ). Therefore the resulting variance of  $Z$  depends on the conditioning event. Moreover, the shape of the density depends on the conditioning event. And I think this should also be the case, if  $X$  and  $Y$  are Gaussian. If  $X$  and  $Y$  are independently uniformly distributed the variance of  $P(Z \mid |X-Y| \leq w)$  is smaller for larger values of  $w$ , because the shape of the resulting variable  $Z$  approaches the shape of the triangular distribution.

If  $w$  is smaller than  $\text{Inf}$ :  
Intuition suggests that  $\text{Var}(Z) > \text{Var}(X)+\text{Var}(Y)$  but does anyone know a formula / approximation for this?

Let  $A = \{|X - Y| \leq w\}$ . Are you asking if  $\text{Var}(Z \mid A) > \text{Var}(X \mid A) + \text{Var}(Y \mid A)$  when  $w$  is finite? That depends on the sign of  $\text{Cov}(X, Y \mid A)$ .

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A). Do you have empirical evidence to suggest that this conditional covariance is always negative?

I am asking, if  $\text{Var}(Z \mid A) > \text{Var}(Z)$ . As written above I do think that the variance (and the distribution) depends on the conditioning event. And this is the link to the second question, if the resulting  $Z$  can be approximated by a Gaussian and what is the approximated variance?