Re: Question in Understanding "converges in probability"

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"Tim" <liting0612@xxxxxxxxx> wrote in message
news:7a7e5925−4667−41bf−8a15−83c0a257646d@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

"The definition says "{Xn} converges in probability to X iff P(|Xn − X|>= å) −−−> 0 for every å > 0". I am wondering if it can be defined without introducing å? Specifically, what's the difference between "P(|Xn − X| > å) −−−> 0 for every å > 0" and "P(|Xn − X| > 0) −−−> 0"? If they are not the same, as a condition which one is stronger ? Please give some clarification. Thanks in advance!"

I am not sure myself. This definition was always a bit not clear to me, but let me give it a shot.

First, the definition in words is, I think, as follows:

Imagine you have some process which generates one random variable after another. Imagine also you have some fixed scalar value. For illustration, lets say X is some point in space, which is the limit we are looking at in this case. Imagine this point X being the center of a circle of radius epsilon where one can make the radius as large or small as we wish.

The definition then says: as more and more r.v.'s are generated, eventually the *probability* that the next one that comes out would be further away from X by some epsilon amount would tend to zero, regardless of what radius one picks.

In other words, the probability that, in the limit, that a random variable will fall outside the circle centered at X will approach zero, no matter how small the radius of the circle is. i.e. eventually the probability that the nth r.v. variable, for very large n, will land inside the circle approaches a certainty for any radius you pick no matter how small.

Now lets look at your alternative definition. You are saying in words: As n gets very large, The probability that the nth r.v. will not land almost at the center of the circle will tend to be zero.

Humm. Well, your definition seems ok, in the limit, but it seems to not show

the convergence process. i.e. You are just saying that the probability that the sequence limit is different from its limit gets to be zero but not shown a mechanism of how this limit progresses. I think this epsilon is important for this part. (it is also important to make student work harder :)

Also notice the definition say for any 'epsilon'. Your definition does not have epsilon at all, so one can't use your definition to say something as follows: the limit of the rv. sequence, will have a probability of it being away from its X by more than 5 is approaching zero. Where I just picked 5, because of the 'for any epsilon' in the definition, I am allowed to do so. right?

In your definition, I could only say: the limit of the rv. sequence have a probability approaching zero of being different from its limit X.

Any way. this is my 2 cents attempt at this late at night. I think this is a good question. You might want to look at convergence almost surely definition, it is a stronger convergence than in probability and might help.

I think if I take another course in statistics, we might study this stuff in more details.

Nasser

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